Part IV

第四部分

Nonlocal Quantum Gravity

非局域量子引力

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26 Nonlocal Gauge Theories Including Quantum Gravity

第 26 章包含量子引力的非局域规范理论

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Abstract

摘要

In this chapter we review nonlocal field theory (infinite derivative field theory). We start with the discussion of the main peculiarities of nonlocal field theory on the example of $d = 4$ scalar ϕ^4 model. The nonlocal ϕ^4 model is ultraviolet finite, unitary, and macrocausal. We consider the model with an infinite number of local scalar fields $\phi_n(x)$ and local interactions among the local fields $\phi_n(x)$. An account of infinite number of local fields $\phi_n(x)$ leads to nonlocal and ultraviolet finite theory. We discuss the generalization of nonlocal ϕ^4 model to the case of abelian and nonabelian gauge field theories. In contrast to the case of ϕ^4 model, nonlocal gauge theories are superrenormalizable but not ultraviolet finite. The introduction of nonlocality allows to make Feynman integrals ultraviolet finite except some finite number of one-loop integrals. Also we review the main results obtained in nonlocal quantum gravity, and we point out that the nonlocal generalization of Einstein gravity can make the theory ultraviolet finite except one-loop level. Especially interesting is nonlocal generalization of renormalizable Stelle gravity which allows to get rid of the problems with negative norm states at least for free graviton propagator. On the example of axial electrodynamics, we discuss gauge theories with γ_5 -anomalies and show how nonlocal gauge field theories can help to make such theories meaningful. Also we consider nonlocal generalization of standard nonsupersymmetric $SU(5)$ Georgi-Glashow GUT and show that it is possible to solve the problems with the proton lifetime and the Weinberg angle without introduction of additional particles in the spectrum. Nonlocal scale Λ responsible for ultraviolet cutoff coincides (up to some factor) with GUT scale $M_{GUT} \approx 3 \cdot 10^{16}$GeV .

本章我们综述非局域场论，也称为无限导数场论。我们以 $d = 4$ 标量 ϕ^4 模型为例，首先讨论非局域场论的主要特性。该非局域 ϕ^4 模型是紫外有限、幺正且宏观因果的。我们考察了包含无限多个局域标量场 $\phi_n(x)$ 且局域场 $\phi_n(x)$ 之间存在局域相互作用的模型。纳入无限多个局域场 $\phi_n(x)$ 后，就得到了非局域的紫外有限理论。我们讨论了将非局域 ϕ^4 模型推广到阿贝尔与非阿贝尔规范场论的情况。与 ϕ^4 模型不同，非局域规范论是超可重整的，但并非紫外有限。引入非局域性可以使费曼积分成为紫外有限的，仅有限数量的单圈积分除外。我们还综述了非局域量子引力中获得的主要结果，并指出爱因斯坦引力的非局域推广可以使该理论在除单圈能级外都是紫外有限的。特别有意思的是可重整斯蒂勒引力的非局域推广，它至少对于自由引力子传播子可以消除负规范态问题。我们以轴子电动力学为例，讨论了存在 γ_5 反常的规范论，并说明非局域规范场论如何帮助让这类理论具有物理意义。我们还考察了标准非超对称 $SU(5)$ 乔治-格拉肖大统一理论的非局域推广，结果表明无需在谱中引入额外粒子即可解决质子寿命和温伯格角问题。负责紫外截断的非局域能标 Λ (相差一个常数因子) 与大统一能标 $M_{GUT} \approx 3 \cdot 10^{16}$GeV 一致。

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Keywords

关键词

Nonlocal field theory . Infinite derivative field theory - Nonlocal quantum gravity

非局域场论。无限导数场理论-非局域量子引力

Introduction

引言

It is well known [1] that local field theories are divided into (super)renormalizable and nonrenormalizable field theories. For renormalizable field theories, ultraviolet divergences are eliminated by the introduction of finite number of local countert-erms. For instance, for the simplest scalar ϕ^4 field theory in d = 4 space-time with the Lagrangian (Here we use the metric (+—).)

众所周知 [1]，定域场论可分为 (超) 可重整场论与不可重整场论。对于可重整场论，紫外发散可通过引入有限个定域抵消项消除。例如，最简单的标量 ϕ^4 场论在 d = 4 时空中拉格朗日量为 (我们这里使用度规 (+—)。)

$$L = \frac{1}{2}\left(\partial^\mu\phi\partial_\mu\phi - m^2\phi^2\right) - \lambda\phi^4 \tag{1}$$

all ultraviolet divergences arising in the calculation of Green's functions in perturbation theory can be removed by the introduction of finite number of local counterterms into the Lagrangian. Namely, the counterterms

微扰论格林函数计算中产生的所有紫外发散，都可以通过向拉格朗日量引入有限个定域抵消项消除。也就是说，抵消项

$$\Delta L = (Z_1 - 1)\frac{1}{2}\left(\partial^\mu\phi\partial_\mu\phi\right) - \frac{\delta m^2}{2}\phi^2 - (Z_2 - 1)\lambda\phi^4 \tag{2}$$

allow to make Green's functions ultraviolet finite in each order of perturbation theory. In $d = 4$ space-time, the most general renormalizable theory describes the interactions of massless spin 1 bosons, spin 1/2 fermions, and spin 0 bosons

可以使格林函数在微扰论的每一阶都是紫外有限的。在 $d = 4$ 时空中，最一般的可重整理论描述了无质量自旋 1 玻色子、自旋 1/2 费米子和自旋 0 玻色子的相互作用

(The spontaneous symmetry breaking of the gauge symmetry allows to make nonzero masses for massless vector bosons.). For instance, the SM (Standard Model) based on gauge group $SU_c(3) \times SU_L(2) \times U(1)$ is renormalizable field theory. For nonrenormalizable field theories like scalar ϕ^6 model in $\mathrm{d} = 4$ spacetime, the number of different counterterms which make Green's functions ultraviolet finite depends on the order of perturbation theory; moreover the number of different counterterms is increasing function of the perturbation theory order. In renormalizable field theories, the predictive power of the theory depends on finite number of unknown parameters - masses and coupling constants - for instance in QED (quantum electrodynamics), the predictions depend on electron mass and fine coupling constant α . In nonrenormalizable field theories, the number of unknown parameters is infinite that makes very weak the predictive power of the theory.

(规范对称性的自发破缺可以让无质量矢量玻色子获得非零质量。) 例如，基于规范群 $SU_c(3) \times SU_L(2) \times U(1)$ 的 SM(标准模型) 是可重整场论。对于不可重整场论，比如 $\mathrm{d} = 4$ 时空中的标量 ϕ^6 模型，能让格林函数成为紫外有限的不同抵消项的数量依赖于微扰论的阶数；且不同抵消项的数量是微扰论阶数的增函数。在可重整场论中，理论的预言能力依赖于有限个未知参数——质量和耦合常数——例如在 QED(量子电动力学) 中，预言结果依赖于电子质量和精细耦合常数 α 。而在不可重整场论中，未知参数的数量是无穷的，这使得理论的预言能力非常弱。

As it was mentioned before, the SM and its extensions like $SU(5)$ GUT and $SU_c(3) \times SU_L(2) \times SU_R(2) \times U(1)$ left-right symmetric model are renormalizable field theories. The situation changes drastically if we consider the gravitational interactions. It is well known that the Einstein gravity is nonrenormalizable field theory at quantum level [2, 3]. Moreover all attempts to make Einstein gravity renormalizable and unitary by the introduction of finite number of additional local fields failed. At present the string theory (See section "String Theory" in this handbook.) is the most perspective approach to make Einstein gravity ultraviolet finite. Another possible way to deal with quantum gravity is the use of nonlocal field theory (infinite derivative field theory) [4, 5] .

如前所述，标准模型及其扩展比如 $SU(5)$ 大统一理论和 $SU_c(3) \times SU_L(2) \times SU_R(2) \times U(1)$ 左右对称模型都是可重整场论。当我们考虑引力相互作用时，情况发生了巨大变化。众所周知，爱因斯坦引力在量子层面是不可重整场论 [2,3]。此外，所有通过引入有限个额外定域场让爱因斯坦引力成为可重整且幺正的尝试都失败了。目前，弦论 (见本手册“弦论”一节) 是让爱因斯坦引力成为紫外有限理论最有前景的方案。另一个研究量子引力的可能方案是使用非定域场论 (无穷导数场论) [4, 5] 。

Many years ago G.V. Efimov proposed to use nonlocal field theory to make the theory ultraviolet finite [4, 6, 7] . G.V. Efimov considered scalar ϕ^4 model and QED in $d = 4$ space-time and proved that the introduction of nonlocal formfactor makes the renormalizable ϕ^4 model ultraviolet finite whereas the nonlocal QED becomes superrenormalizable field theory. It is well known that the simplest way to deal with ultraviolet divergent Feynman diagrams is the introduction of the regularization. For instance, in Pauli-Villars regularization [8] for scalar ϕ^4 field theory, the replacement of scalar propagator

多年前 G.V. 叶菲莫夫就提出使用非定域场论来获得紫外有限的理论 [4, 6, 7] 。G.V. 叶菲莫夫研究了 $d = 4$ 时空中的标量 ϕ^4 模型和量子电动力学，证明了引入非定域形状因子可以让可重整的 ϕ^4 模型变为紫外有限，而非定域量子电动力学会成为超可重整场论。众所周知，处理紫外发散费曼图最简单的方法就是引入正则化。例如，在标量 ϕ^4 场论的泡利-维拉尔正则化 [8] 中，需要替换标量传播子

$$D\left(p^2\right) \to D_{reg}\left(p^2\right) = \frac{1}{m^2 - p^2 - i\varepsilon} - \frac{1}{M^2 - p^2 - i\varepsilon} = \frac{V_{PV}\left(p^2\right)}{m^2 - p^2 - i\varepsilon} \tag{3}$$

makes all Feynman integrals ultraviolet finite except vacuum diagrams and one-loop correction to the scalar propagator. The main drawback of Pauli-Villars regularization is that the introduction of the second term $-\frac{1}{M^2-p^2-i\varepsilon}$ in (3) is equivalent to the introduction of negative norm state with a mass M in the spectrum. The ϕ^4 model with Pauli-Villars propagator (3) is local and unitary, but it contains negative norm states that makes impossible reasonable physical interpretation of such model. The main idea of nonlocal field theory [4] consists in the replacement of the local propagator $D\left(p^2\right)$ to nonlocal propagator, namely,

> 使得除真空图和标量传播子的单圈修正外，所有费曼积分都是紫外有限的。泡利-维拉斯正规化的主要缺点是，式 (3) 中第二项 $-\frac{1}{M^2-p^2-i\varepsilon}$ 的引入等价于在谱中引入一个质量为 M 的负范数态。带有泡利-维拉斯传播子 (3) 的 ϕ^4 模型是定域且幺正的，但它包含负范数态，导致无法对该模型给出合理的物理解释。非定域场论 [4] 的核心思想是将定域传播子 $D\left(p^2\right)$ 替换为非定域传播子，即:

$$D\left(p^2\right) \to D_{nl}\left(p^2\right) = \frac{V\left(p^2\right)}{m^2 - p^2 - i\varepsilon}, \tag{4}$$

where $V(z)$ (nonlocal formfactor) is entire function in complex z -plane decreasing in the euclidean region $V\left(p^2\right) \to 0$ at $p^2 \to -\infty$, for instance $V\left(p^2\right) = \exp\left[\frac{(p^2-m^2)}{\Lambda^2}\right]$. Here Λ is nonlocal scale, and it is assumed that $V\left(p^2\right) \to 1$ at $\Lambda \to \infty$, i.e., in the infinite Λ limit, we obtain local ϕ^4 model (We assume that formfactor $V\left(p^2\right)$ implicitly depends on nonlocal scale Λ .). Other interesting formfactor is

> 其中 $V(z)$ (非定域形状因子) 是复 z 平面上的整函数，在欧几里得区域 $V\left(p^2\right) \to 0$ 中 $p^2 \to -\infty$ 处衰减，例如 $V\left(p^2\right) = \exp\left[\frac{(p^2-m^2)}{\Lambda^2}\right]$。此处 Λ 是非定域能标，假设当 $\Lambda \to \infty$ 时满足 $V\left(p^2\right) \to 1$，即在 Λ 取无穷极限时我们得到定域 ϕ^4 模型 (我们认为形状因子 $V\left(p^2\right)$ 隐含依赖于非定域能标 Λ)。另一个常用的形状因子是

$$V\left(p^2\right) = V_{PV}\left(p^2\right)\left[1 - \sin\left(1 - \frac{p^2}{\Lambda^2}\right) \cdot \left(1 - \frac{p^2}{\Lambda^2}\right)^{-1}\right], \ \Lambda = M. \tag{5}$$

In euclidean field theory at $p^2 < 0$, Feynman integrals have damping formfactor $V\left(p^2\right)$, and as a consequence Feynman integrals become ultraviolet finite. In the limit $\Lambda \to \infty D_{nl}\left(p^2\right) \to D\left(p^2\right)$ and we reproduce local field theory. Both local renormalizable field theories and nonlocal field theories are Poincare invariant and unitary. The crucial difference among them is that local renormalizable field theories are local and microcausal while nonlocal field theories are only macrocausal. As it was mentioned before, it is reasonable to assume that the world without gravity is described by the renormalizable SM or its renormalizable extensions. Such theories are well defined except maybe the problems with Landau pole singularities (In the SM Landau pole singularity for $U(1)$, gauge subgroup takes place at energies much higher the Planck scale $M_{PL} = 1.2 \times 10^{19}$GeV where the effects of quantum gravity are essential.). The situation changes drastically in the world with gravity. A possible way to overcome problems with nonrenormalizability of quantum gravity is the use of nonlocal gravity [9] to make the theory renormalizable or even superrenormalizable and escape the problems with negative norm states. Very important question arises - what about the relation between nonlocal scale Λ and the Planck scale M_{PL} ? We expect that $\Lambda \leq M_{PL}$ because in the opposite case, the ultraviolet cutoff Λ can't help to make tree-level Feynman S-matrix elements well behaved at large energies. The case with $\Lambda \ll M_{PL}$ is possible; however the value $\Lambda \sim M_{PL}$ looks more natural. It should be stressed that at present the single serious motivation in favor of nonlocal field theory is the fact that Einstein gravity is nonrenormalizable field theory and its nonlocal generalization can solve the problems with the nonrenormalizability.

在欧几里得场论中，当 $p^2 < 0$ 时，费曼积分带有衰减形状因子 $V(p^2)$，因此费曼积分变为紫外有限。在 $\Lambda \to \infty D_{nl}(p^2) \to D(p^2)$ 极限下我们可以还原得到定域场论。定域可重整场论和非定域场论都满足庞加莱不变性和幺正性。二者的关键区别在于: 定域可重整场论是定域且微观因果的，而非定域场论仅满足宏观因果性。如前所述，我们有合理的理由认为，无引力的世界由可重整的标准模型或其可重整扩展描述。除朗道极点奇点问题外，这类理论的定义是良好的 (在标准模型中，$U(1)$ 规范子群的朗道极点出现在远高于普朗克能标 $M_{PL} = 1.2 \times 10^{19}$GeV 的能量处，而量子引力效应在该能标已经变得重要)。在包含引力的世界中，情况发生了剧烈变化。解决量子引力不可重整问题的一种可能方案是使用非定域引力 [9]，它可以让理论变为可重整甚至超可重整，并且避免负范态问题。由此产生了一个非常重要的问题: 非定域能标 Λ 和普朗克能标 M_{PL} 之间是什么关系？我们认为 $\Lambda \leq M_{PL}$，因为若非如此，紫外截断 Λ 无法帮助树级费曼 S 矩阵元在高能区保持良好行为。$\Lambda \ll M_{PL}$ 的情况是可能存在的；但 $\Lambda \sim M_{PL}$ 的取值看起来更自然。需要强调的是，目前支持非定域场论唯一可靠的动机是: 爱因斯坦引力是不可重整场论，而它的非定域推广可以解决不可重整性的问题。

In this chapter we review nonlocal field theories including nonlocal gauge theories and nonlocal quantum gravity. We discuss gauge theories with γ_5 -anomalies and show how nonlocal gauge field theories can help to make such theories meaningful. We also consider nonlocal generalization of standard nonsupersymmetric $SU(5)$ Georgi-Glashow GUT and show that it is possible to solve the problems with the proton lifetime and the Weinberg angle without introduction of additional particles in the spectrum of the theory. For nonlocal $SU(5)$ Georgi-Glashow GUT nonlocal scale Λ responsible for ultraviolet cutoff coincides (up to some factor) with GUT scale M_{GUT} . In the simplest nonlocal modification of the $SU(5)$ GUT model $M_{GUT} \approx 3 \cdot 10^{16}$GeV.

本章我们回顾非定域场论，包括非定域规范理论与非定域量子引力。我们讨论存在 γ_5 反常的规范理论，说明非定域规范场论如何帮助这类理论具备物理意义。我们还考虑了标准非超对称 $SU(5)$ 乔吉-格拉肖大统一理论的非定域推广，证明无需在理论谱中引入额外粒子即可解决质子寿命和温伯格角问题。对于非定域 $SU(5)$ 乔吉-格拉肖大统一理论，负责紫外截断的非定域能标 Λ (相差一个常数因子) 与大统一能标 M_{GUT} 一致。在 $SU(5)$ 大统一模型最简单的非定域修正中 $M_{GUT} \approx 3 \cdot 10^{16}$GeV.

The chapter is organized as follows. In the next section, we discuss the main peculiarities of nonlocal field theory on the example of $\mathrm{d} = 4\phi^4$ model. We show that nonlocal $\mathrm{d} = 4\phi^4$ model is unitary and macrocausal. In section "Local Field Theory with Infinite Number of Local Fields as Origin of Nonlocality" we consider the model with an infinite number of local scalar fields $\phi_n(x)$ and local interactions with higher-order derivatives for each local field $\phi_n(x)$. An account of infinite number of local fields leads to nonlocal and ultraviolet finite theory. In section "Nonlocal Gauge Theories" we discuss the generalization of nonlocal $d = 4\phi^4$ model to the case of abelian and nonabelian gauge field theories. In contrast to the case of $\mathrm{d} = 4\phi^4$ model, nonlocal gauge theories are superrenormalizable but not ultraviolet finite. The introduction of nonlocality allows to make Feynman integrals ultraviolet finite except some finite number of one-loop integrals. In section "Nonlocal Gravity" we review the main results obtained in nonlocal quantum gravity. The introduction of nonlocality makes the theory ultraviolet finite except one-loop level. Also we point out that nonlocal generalization of renormalizable Stelle gravity allows to get rid of the problems with negative norm states at least for free graviton propagator. In section "Some Applications of Nonlocal Field Theory" we study two possible applications of nonlocal field theory. As the first example, we consider γ_5 -anomalous field theory - axial QED. We show that nonlocal generalization of axial QED is superrenormalizable theory describing the interaction of massive vector boson with massless fermion. As the second example, we consider nonlocal generalization of standard nonsupersymmetric $SU(5)$ Georgi-Glashow GUT and show that it is possible to solve the problems with the proton lifetime and the Weinberg angle without introduction of additional particles in the spectrum of the theory. Nonlocal scale Λ responsible for ultraviolet cutoff coincides (up to some factor) with GUT scale M_{GUT} . In the simplest nonlocal modification of the $SU(5)$ GUT $M_{GUT} \approx 3 \cdot 10^{16}$GeV . Section "Conclusions" contains concluding remarks.

本章结构安排如下: 下一节我们以 d = $4\phi^4$ 模型为例讨论非定域场论的主要特性，说明非定域 d = $4\phi^4$ 模型满足幺正性与宏观因果性。在“作为非定域起源的含无穷多定域场的定域场论”一节中，我们研究含无穷多定域标量场 $\phi_n(x)$ 且每个定域场 $\phi_n(x)$ 都存在高阶导数定域相互作用的模型。计入无穷多定域场后可得到非定域且紫外有限的理论。在“非定域规范理论”一节中，我们讨论非定域 $d = 4\phi^4$ 模型到阿贝尔与非阿贝尔规范场论的推广。与 d = $4\phi^4$ 模型不同，非定域规范理论是超可重整化的，但并非紫外有限的。引入非定域性可以让费曼积分紫外有限，仅有限数目的单圈积分除外。在“非定域引力”一节中，我们回顾非定域量子引力得到的主要结果: 引入非定域性后，除单圈阶外理论都是紫外有限的；我们还指出，可重整化施泰勒引力的非定域推广至少在自由引力子传播子层面可以解决负规范态问题。在“非定域场论的若干应用”一节中，我们研究非定域场论的两个可能应用: 第一个例子我们研究存在 γ_5 反常的场论——轴向量子电动力学，证明轴向量子电动力学的非定域推广是描述有质量矢量玻色子与无质量费米子相互作用的超可重整化理论。第二个例子我们考虑标准非超对称 $SU(5)$ 乔吉-格拉肖大统一理论的非定域推广，证明无需在理论谱中引入额外粒子即可解决质子寿命和温伯格角问题。负责紫外截断的非定域能标 Λ (相差一个常数因子) 与大统一能标 M_{GUT} 一致。在 $SU(5)$ 大统一最简单的非定域修正 $M_{GUT} \approx 3 \cdot 10^{16}$GeV 中。“结论”一节给出总结性评述。

Nonlocal Φ^4 Model

非局域 Φ^4 模型

The nonlocal analog of the local Lagrangian (1) has the form:

局域拉格朗日量 (1) 的非局域类比形式如下:

$$L_{nl} = \frac{1}{2}\left(\partial^\mu \phi V^{-1}\left(-\partial^\mu \partial_\mu\right)\partial_\mu \phi - m^2 \phi V^{-1}\left(-\partial^\mu \partial_\mu\right)\phi\right) - \lambda \phi^4. \tag{6}$$

Here the formfactor $V(p^2)$ is an entire function in complex p^2 plane and $V(p^2) \to 0$ at $p^2 \to -\infty$ that leads to the ultraviolet finite Feynman diagrams in euclidean space-time. G.V. Efimov has considered [4, 6, 7] nonlocal scalar ϕ^4 model with formfactor $V(z)$ satisfying the following requirements:

此处形状因子 $V(p^2)$ 是复 p^2 平面上的整函数, 且在 $p^2 \to -\infty$ 处满足 $V(p^2) \to =0$, 这使得欧几里得时空中的费曼图都是紫外有限的。G.V. 叶菲莫夫研究了满足以下条件、形状因子为 $V(z)$ 的 [4, 6, 7] 非局域标量 ϕ^4 模型:

1. $V(z)$ is the entire function in complex z plane of the growth $\rho \geq 1/2$, i.e. $V(z) \leq \ C \cdot \exp\left(b|z|^{\rho}\right)$

1. $V(z)$ 是复 z 平面上增长阶为 $\rho \geq 1/2$ 的整函数，即 $V(z) \leq \ C \cdot \exp\left(b|z|^{\rho}\right)$

2. $V(z) = O\left(z^{-2}\right)$ at $\operatorname{Re} z \to -\infty$

2. 在 $\operatorname{Re} z \to -\infty$ 处的 $V(z) = O\left(z^{-2}\right)$

3. $V(z) = V^{*}(z^{*})$
4. $V\left(m^2\right) = 1$
5. $V(z) > 0$ for real z .

5. 对于实 z ，$V(z) > 0$ 成立。

G.V. Efimov has proved that nonlocal model ϕ^4 model is unitary and macrocausal in perturbation theory. The main idea of the unitarity proof is based on the so-called formfactor quantization. The entire function $V\left(p^2\right)$ of the growth ρ can be presented in the form:

G.V. 叶菲莫夫已经证明，微扰论中非局域模型 ϕ^4 模型是幺正的且满足宏观因果性。幺正性证明的核心思想基于所谓的形状因子量子化。增长阶为 ρ 的整函数 $V\left(p^2\right)$ 可写为如下形式:

$$V\left(p^2\right) = \sum_{n=0}^{n=\infty} v_n\left(p^2 m^{-2} - 1\right)^n, \tag{7}$$

where $v_n \sim [\Gamma(n/\rho)]^{-1}$ at $n \to \infty$. The main trick consists in the "regularization" of the $V\left(p^2\right)$ formfactor, namely,

其中在 $n \to \infty$ 处 $v_n \sim [\Gamma(n/\rho)]^{-1}$ 。核心技巧在于对 $V\left(p^2\right)$ 形状因子做“正则化”，即

$$V\left(p^2\right) \to V^{\delta}\left(p^2\right), \tag{8}$$

where

其中

$$V^{\delta}\left(p^2\right) = \sum_{n=0}^{n=\infty} v_n\left(p^2 m^{-2} - 1\right)^n \left[\Pi_{j=1}^{j=n+n_0}\left(1 - \delta j^{-\sigma}\left(p^2 m^{-2} - 1\right)\right)\right]^{-1}. \tag{9}$$

Here $n_0 \geq 2$ and $\sigma\rho < 1$. The regularized propagator $D^{\delta}\left(p^2\right) = \left(m^2 - \ p^2 - i\varepsilon\right)^{-1} V^{\delta}\left(p^2\right)$ is meromorphic function in complex p^2 plane, and it can be represented in the form:

此处 $n_0 \geq 2$ 且 $\sigma\rho < 1$ 。正则化传播子 $D^{\delta}\left(p^2\right) = \left(m^2 - \ p^2 - i\varepsilon\right)^{-1} V^{\delta}\left(p^2\right)$ 是复 p^2 平面上的亚纯函数，可表示为:

$$D^{\delta}\left(p^{2}\right)=\left(m^{2}-p^{2}-i\varepsilon\right)^{-1}+\sum_{j=1}^{j=\infty}(-1)^{j}A_{j}(\delta)\left(m_{j}^{2}(\delta)-p^{2}-i\varepsilon\right)^{-1}. \tag{10}$$

Here

此处

$$m_{j}^{2}(\delta)=m^{2}\left(1+j^{\sigma}\delta^{-1}\right) \tag{11}$$

and

且

$$A_{j}(\delta)=\sum_{n=\max[0,j-n_{0}]}v_{n}\left(j^{\sigma}\delta^{-1}\right)^{n}\Pi_{k=1,k\neq j}^{n+n_{0}}\left[1-\left(jk^{-1}\right)^{\sigma}\right]^{-1}. \tag{12}$$

The following identities are valid:

下列恒等式成立:

$$\sum_{j=0}^{j=\infty}(-1)^{j}A_{j}(\delta)\left[m_{j}^{2}(\delta)\right]^{s}=0 \tag{13}$$

for $s=0,1,\ldots,n_{0}$. In the limit of the regularization removing $\delta\to 0$ for any finite value of $p^{2}D^{\delta}\left(p^{2}\right)\to D\left(p^{2}\right)$.

对于 $s=0,1,\ldots,n_{0}$ 。在移除正则化的极限 $\delta\to 0$ 下，对于 $p^{2}D^{\delta}\left(p^{2}\right)\to D\left(p^{2}\right)$ 的任意有限值都成立。

We can consider the regularization (9) as generalization of Pauli-Villars regularization (3). The difference between Paul-Villars regularization (3) and the generalization (9) is the following. For Pauli-Villars regularization (3)

我们可以将正则化 (9) 看作泡利-维拉尔正则化 (3) 的推广。泡利-维拉尔正则化 (3) 和推广 (9) 的区别如下。对于泡利-维拉尔正则化 (3)

$$D_{reg}\left(p^{2}\right)\to D\left(p^{2}\right)=\frac{1}{m^{2}-p^{2}-i\varepsilon} \tag{14}$$

in the limit of the regularization removing $M\to\infty$. To make Feynman integrals ultraviolet finite, we have to introduce counterterms (2) in each order of perturbation theory. For the regularized nonlocal propagator $D^{\delta}\left(p^{2}\right)$

在移除正则化的极限 $M\to\infty$ 下。为了使费曼积分紫外有限，我们需要在微扰论的每一阶引入抵消项 (2)。对于正则化非局域传播子 $D^{\delta}\left(p^{2}\right)$

$$D^{\delta}\left(p^{2}\right) \rightarrow \frac{V\left(p^{2}\right)}{m^{2}-p^{2}-i \varepsilon} \text { for } \delta \rightarrow 0. \tag{15}$$

Due to the presence of nonlocal formfactor $V\left(p^{2}\right)$, all Feynman integrals are ultraviolet finite in euclidean space-time in each order of perturbation theory. As a consequence we don't have to introduce local counterterms (2) to make the theory ultraviolet finite.

由于存在非局部形状因子 $V\left(p^{2}\right)$，所有费曼积分在欧几里得时空的任意阶微扰论中都是紫外有限的。因此我们无需引入局部 counterterms(2) 使理论成为紫外有限。

The regularized Lagrangian

正则化拉格朗日量

$$L_{nl,\delta}=\frac{1}{2} \partial^{\mu} \phi\left(V^{\delta}\left(-\partial^{\mu} \partial_{\mu}\right)\right)^{-1} \partial_{\mu} \phi-\frac{1}{2} m^{2} \phi\left(V^{\delta}\left(-\partial^{\mu} \partial_{\mu}\right)\right)^{-1} \phi-\lambda \phi^{4} \tag{16}$$

is equivalent to the Lagrangian

等价于如下拉格朗日量

$$L_{\delta}=L_{0, \delta}+L_{\text {int }}, \tag{17}$$

where

其中

$$L_{0, \delta}=\sum_{j=0}^{j=\infty} \frac{1}{2}(-1)^{j}\left[\partial^{\mu} \phi_{j}^{\delta} \partial_{\mu} \phi_{j}^{\delta}-m_{j}^{2}(\delta) \phi_{j}^{\delta} \phi_{j}^{\delta}\right], \tag{18}$$

$$L_{\text {int }}=-\lambda\left(\Phi^{\delta}\right)^{4}, \tag{19}$$

$$\Phi^{\delta}=\sum_{j=0}^{j=\infty} \phi_{j}^{\delta} \tag{20}$$

The regularized Lagrangian L_{δ} describes the self-interaction of the infinite number of scalar fields ϕ_{j}^{δ} with masses $m_{j}(\delta)$. The quantization of such system is straightforward, namely, we postulate the following canonical relations:

正则化拉格朗日量 L_{δ} 描述了无穷多个质量为 $m_{j}(\delta)$ 的标量场 ϕ_{j}^{δ} 的自相互作用。该系统的量子化十分直接，我们假定如下正则关系:

$$\left[\phi_{j}^{\delta}(0, \vec{x}), \phi_{j^{\prime}}^{\delta}\left(0, \vec{x}^{\prime}\right)\right]=\left[\pi_{j}^{\delta}(0, \vec{x}), \pi_{j^{\prime}}^{\delta}\left(0, \vec{x}^{\prime}\right)\right]=0, \tag{21}$$

$$\left[\phi_{j}^{\delta}(0, \vec{x}), \pi_{j^{\prime}}^{\delta}\left(0, \vec{x}^{\prime}\right)\right]=i \delta_{j j^{\prime}} \delta\left(\vec{x}-\vec{x}^{\prime}\right), \tag{22}$$

where

其中

$$\pi_j^\delta\left(\vec{x},t\right)=(-1)^j\frac{d\phi_j^\delta\left(\vec{x},t\right)}{dt}, \tag{23}$$

As a consequence of the commutation relations (21,22), we find that the Hamiltonian corresponding to noninteracting Lagrangian $L_{o,\delta}$ can be written in the form:

由对易关系 (21,22) 可得，对应非相互作用拉格朗日量 $L_{o,\delta}$ 的哈密顿量可写为:

$$H_0^\delta=\sum_{j=0}^{\infty}(-1)^j\int d\vec{p}\,\omega_{j\vec{p}}^\delta d_{j\vec{p}}^+ d_{j\vec{p}}, \tag{24}$$

where $\omega_{j\vec{p}}^\delta=\sqrt{\vec{p}^2+m_j^2(\delta)}$ and operators $d_{j\vec{p}}$ satisfy the commutation relations

其中 $\omega_{j\vec{p}}^\delta=\sqrt{\vec{p}^2+m_j^2(\delta)}$ 与算符 $d_{j\vec{p}}$ 满足对易关系

$$\left[d_{j\vec{p}},d_{j'\vec{p}'}\right]=\left[d_{j\vec{p}+}^+,d_{j'\vec{p}'}^+\right]=0, \tag{25}$$

$$\left[d_{j\vec{p}},d_{j'\vec{p}'}^+\right]=\left[d_{j\vec{p}+},d_{j'\vec{p}'}^+\right]=(-1)^j\delta_{jj'}\delta\left(\vec{p}-\overrightarrow{p'}\right), \tag{26}$$

The Hamiltonian (24) describes free scalar particles with masses $m_j^2(\delta)$ for $j=\ 1,2,3\ldots$ and $m_{j=0}^2(\delta)=m^2$ for $j=0$. The fields ϕ_j^δ with even j correspond to scalar particles with positive metric, while the fields ϕ_j^δ with odd j correspond to scalar particles with indefinite metric. For $\delta\neq 0$ the physical spectrum contains states with positive and negative metric. In the limit $\delta\to 0$ masses of all ghost states $m_i(\delta)\to\infty$. We characterize physical states by definite finite energy. As a consequence in the limit of regularization removing $\delta\to 0$, all states with finite energy are the states with positive energy, namely, such states consist of scalar particles with a mass m .

哈密顿量 (24) 描述了质量分别为 $m_j^2(\delta)$ (对应 $j=\ 1,2,3\ldots$) 和 $m_{j=0}^2(\delta)=m^2$ (对应 $j=0$) 的自由标量粒子。j 为偶数的场 ϕ_j^δ 对应正度规的标量粒子，j 为奇数的场 ϕ_j^δ 对应不定度规的标量粒子。对于 $\delta\neq 0$ ，物理谱包含正度规态和负度规态。在极限 $\delta\to 0$ 下，所有鬼态 $m_i(\delta)\to\infty$ 的质量趋于无穷。我们用确定的有限能量表征物理态，因此在移除正则化的极限 $\delta\to 0$ 下，所有有限能量态都是正能量态，即这类态都由质量为 m 的标量粒子构成。

Unitarity

幺正性

The unitarity of nonlocal ϕ^4 model has been proved in Refs. [5,7] within perturbation theory. Recently these results have been confirmed in Refs. [10,11]; see also F. Briscese this handbook. Intuitively it is obvious. Really, the spectrum of the regularized model (17-20) contains infinite number of scalar particles with definite and indefinite metrics and with masses $m_j(\delta)$. The regularized model is unitary because the interaction Lagrangian is Hermitian. The spectrum of the regularized model (17-20) contains the j odd states with indefinite metric. If we consider the states with finite energy, we find that in the limit $\delta \to 0$, all states with $j \neq 0$ particles will have infinite energy and only scalar particles with $j = 0$ and a mass m survive in the spectrum. In other words in the limit $\delta \to 0$, the spectrum contains only scalar particles with a mass m and the S -matrix is unitary. It should be stressed that it is not obvious that for the regularized S-matrix S^δ the limit $\delta \to 0$ exists; see [5, 7] .

非局域 ϕ^4 模型的幺正性已在参考文献 [5,7] 中通过微扰论得到证明。近来这些结果又得到了参考文献 [10,11] 的验证；另见本手册中 F. Briscese 的相关内容。这在直观上是显然的。实际上，正规化模型 (17-20) 的谱包含无穷多标量粒子，分为定度量和不定度量两类，质量为 $m_j(\delta)$ 。该正规化模型是幺正的，因为相互作用拉格朗日量是厄米的。正规化模型 (17-20) 的谱包含具有不定度量的 j 奇宇称态。若我们考虑有限能量的态，会发现在 $\delta \to 0$ 极限下，所有包含 $j \neq 0$ 粒子的态都将具有无穷大能量，谱中仅剩下质量为 m 、带有 $j = 0$ 的标量粒子。换句话说，在 $\delta \to 0$ 极限下，谱中仅包含质量为 m 的标量粒子，且 S 矩阵是幺正的。需要强调的是，对于正规化 S 矩阵 S^δ ， $\delta \to 0$ 极限的存在性并非显然，参见 [5, 7] 。

Causality

因果性

To understand the meaning and consequences of causality, consider the following example [12]. Suppose real scalar field $\phi(x^0, \vec{x})$ evolves by means of differential operator $F(\Box)$ in the presence of a source $j(x^0, \vec{x})$ and the following differential equation is valid:

为理解因果性的含义与推论，我们考察如下例子 [12]。假设实标量场 $\phi(x^0, \vec{x})$ 通过微分算符 $F(\Box)$ 在源 $j(x^0, \vec{x})$ 存在的情况下演化，满足如下微分方程:

$$F(\Box)\phi(x^0, \vec{x}) = -j(x^0, \vec{x}). \tag{27}$$

The solution of the equation (27) has the form:

方程 (27) 的解形式为:

$$\phi(x^0, \vec{x}) = \phi_0(x^0, \vec{x}) + i\int dy^0 d\vec{y} G(x^0 - y^0, \vec{x} - \vec{y}) j(y^0, \vec{y}), \tag{28}$$

where $\phi_0(x^0, \vec{x})$ is the solution of the homogeneous equation

其中 $\phi_0(x^0, \vec{x})$ 是齐次方程的解

$$F(\Box)\phi(x^0, \vec{x}) = 0 \tag{29}$$

and Green’s function $G(x^0 - y^0, \vec{x} - \vec{y})$ satisfies the equation:

且格林函数 $G\left(x^0-y^0,\vec{x}-\vec{y}\right)$ 满足方程:

$$F(\Box)\,G\left(x^0-y^0,\vec{x}-\vec{y}\right)=i\delta\left(x^0-y^0\right)\delta\left(\vec{x}-\vec{y}\right). \tag{30}$$

The solution (28) is said to be causal if Green’s function $G\left(x^0-y^0,\vec{x}-\vec{y}\right)$ can be chosen such that

若格林函数 $G\left(x^0-y^0,\vec{x}-\vec{y}\right)$ 可被选为满足下述条件，则称解 (28) 是因果的:

$$G\left(x^0,\vec{x}\right)=0 \text{ for } x^0<0. \tag{31}$$

The meaning of the causality condition (31) is that the system cannot respond to an interaction before interaction was turned on. The causality definition (31) takes place for both relativistic and nonrelativistic systems. The stronger relativistic generalization of causality (31) is the following. The relativistic Green function is microcausal and local [1] if it obeys the condition (31) and besides it vanishes for $x^2<0$, i.e.,

因果性条件 (31) 的含义是系统在相互作用开启前不会对其产生响应。因果性定义 (31) 对相对论和非相对论系统均成立。因果性 (31) 更强的相对论推广如下: 若相对论格林函数满足条件 (31) 且此外在 $x^2<0$ 时为零，即满足下式，则它是微观因果的且局域 [1]:

$$G\left(x^0,\vec{x}\right)=0 \text{ if } \left|x^0\right|<\left|\vec{x}\right|. \tag{32}$$

Such Green’s functions are often denoted as retarded. In local ϕ^4 theory $F(\Box)=\Box+m^2$ and the retarded Green function is [1]

这类格林函数通常被标记为推迟格林函数。在局域 ϕ^4 理论中 $F(\Box)=\Box+m^2$ ，推迟格林函数为 [1]

$$\begin{aligned} D^{\text{ret}}(x) &= \frac{1}{(2\pi)^4}\int\frac{e^{-ikx}}{m^2-k^2-i\varepsilon k_0}d^4k \\ &= \frac{1}{2\pi}\theta\left(x^0\right)\left[\delta\left(x^2\right)-\theta\left(x^2\right)\frac{m}{2\sqrt{x^2}}J_1\left(m\sqrt{x^2}\right)\right]. \end{aligned} \tag{33}$$

The retarded Green function (33) can be expressed in terms of free scalar fields, namely,

推迟格林函数 (33) 可以用自由标量场表示，即:

$$D^{\text{ret}}(x-y)=i\theta\left(x^0-y^0\right)<0\left[\phi(x),\phi(y)\right]|0>, \tag{34}$$

where $[\phi(x),\phi(y)]=\phi(x)\phi(y)-\phi(y)\phi(x)$ and $|\,0>$ is the vacuum state. The fact that $D^{\text{ret}}(x)=0$ at $x^2<0$ or at $x^0<0$ is a consequence of the formulae (33,34). In nonlocal scalar field theory $F(\Box)=\left(\Box+m^2\right)V^{-1}(-\Box)$, and the retarded function has the form:

其中 $[\phi(x),\phi(y)]=\phi(x)\phi(y)-\phi(y)\phi(x)$ 和 $|\,0>$ 是真空态。$D^{\text{ret}}(x)=0$ 在 $x^2<0$ 或 $x^0<0$ 处为零是公式 (33,34) 的推论。在非局域标量场论中 $F(\Box)=\left(\Box+m^2\right)V^{-1}(-\Box)$ ，推迟函数形式为:

$$D_{nl}^{ret}(x) = \frac{1}{(2\pi)^4} \int \frac{V\left(k^2\right) e^{-ikx}}{m^2 - k^2 - i\varepsilon k^0} d^4k = D^{ret}(x) + D_{ac}^{ret}(x), \tag{35}$$

where

其中

$$D_{ac}^{\text{ret}}(x) = \frac{1}{(2\pi)^4} \int \frac{\left(V\left(k^2\right) - 1\right) e^{-ikx}}{m^2 - k^2 - i\varepsilon k^0} d^4k. \tag{36}$$

Note that due to normalization condition $V\left(m^2\right) = 1$, the integral in formula (36) does not contain singularity at $m^2 - k^2 - i\varepsilon k^0 = 0$. As a consequence the function $D_{ac}^{ret}(x)$ depends only on x^2 . The $D_{ac}^{ret}(x)$ is not causal, i.e., $D_{ac}^{ret}(x) \neq 0$ at $x^2 < 0$ or $x^0 < 0$ [12, 13] . Remember that nonlocal formfactor $V\left(p^2\right)$ depends implicitly on nonlocal scale Λ and $V\left(p^2\right) \to 1$ at $\Lambda \to \infty$. As a consequence acausal Green function $D_{ac}^{ret}(x) \to 0$ at $y \equiv -x^2\Lambda^2 \to \infty$. It means that at macroscopical distances $|x^2| \gg \frac{1}{\Lambda^2}$, causality restores and essential violation of causality takes place only at the distances $l_0 = O\left(\Lambda^{-1}\right)$. In Ref. [13] it was shown that $D_{ac}^{\text{ret}}(x)$ decreases as $D_{ac}^{\text{ret}}(x) \to \exp\left(-a \parallel x \parallel^{\beta}\right)$ at $x \to \infty$ outside the future cone V^+ ; see also [12] and S. Giassari this handbook. Here $a > 0, \beta < \frac{2\rho}{2\rho - 1}, \parallel x \parallel = \sqrt{|x^2|}$ and the $\rho \geq 1/2$ is the growth order of formfactor $V(z)$. It is interesting to note [6] that for nonlocal formfactors of the growth $\rho = 1/2$ the acausal function $D_{ac}^{\text{ret}}(x) = 0$ for $x^2 < 0$ at $-x^2 > l_0^2$, where l_0 depends on the formfactor.

请注意，由于归一化条件 $V\left(m^2\right) = 1$ ，公式 (36) 中的积分在 $m^2 - k^2 - i\varepsilon k^0 = 0$ 处不含奇点，因此函数 $D_{ac}^{ret}(x)$ 仅依赖于 x^2 。$D_{ac}^{ret}(x)$ 不满足因果性，即当 $x^2 < 0$ 或 $x^0 < 0$ [12, 13] 时满足 $D_{ac}^{ret}(x) \neq 0$ 。注意非局部形状因子 $V\left(p^2\right)$ 隐含依赖于非局部标度 Λ ，且在 $\Lambda \to \infty$ 处满足 $V\left(p^2\right) \to 1$ ，因此非因果格林函数 $D_{ac}^{ret}(x) \to 0$ 在 $y \equiv -x^2\Lambda^2 \to \infty$ 处成立。这意味着当距离为宏观尺度 $|x^2| \gg \frac{1}{\Lambda^2}$ 时，因果性恢复，因果性的本质破坏仅发生在距离为 $l_0 = O\left(\Lambda^{-1}\right)$ 处。文献 [13] 表明，在未来光锥 V^+ 外，当 $x \to \infty$ 时 $D_{ac}^{\text{ret}}(x)$ 随 $D_{ac}^{\text{ret}}(x) \to \exp\left(-a \parallel x \parallel^{\beta}\right)$ 衰减；另见 [12] 与本手册中 S. Giassari 的内容。此处 $a > 0, \beta < \frac{2\rho}{2\rho - 1}, \parallel x \parallel = \sqrt{|x^2|}$ ，且 $\rho \geq 1/2$ 是形状因子 $V(z)$ 的增长阶。值得注意的是 [6]，对于增长阶为 $\rho = 1/2$ 的非局部形状因子，当 $-x^2 > l_0^2$ 处 $x^2 < 0$ 时非因果函数 $D_{ac}^{\text{ret}}(x) = 0$ ，其中 l_0 依赖于该形状因子。

To summarize, in nonlocal field theory, we have acausal behavior of retarded Green's functions; however acausality strongly decreases at distances larger nonlocal radius $r_{nl} \equiv \frac{1}{\Lambda}$ so at macroscopical distances we have causal theory. For nonlocal formfactors of the growth $\rho = 1/2$, the effects of acausality take place only at small distances. Therefore the nonlocal field theory with nonlocal formfactors of the $\rho = 1/2$ growth is closest to local field theory.

综上，在非局部场论中，推迟格林函数存在非因果行为；然而在距离大于非局部半径 $r_{nl} \equiv \frac{1}{\Lambda}$ 时非因果性显著衰减，因此宏观距离下该理论满足因果性。对于增长阶为 $\rho = 1/2$ 的非局部形状因子，非因果效应仅发生在小尺度距离上。因此，带有 $\rho = 1/2$ 增长阶非局部形状因子的非局部场论最接近局部场论。

Ultraviolet Finiteness or How to Calculate Feynman Integrals

紫外有限性，或如何计算费曼积分

An entire function $V\left(p^2\right)$ has an essential singularity at complex infinity making the use of Wick rotation unjustified. It means that nonlocal Feynman integrals are not well defined in Minkowski space-time [5, 7, 10, 11] , while the corresponding integrals are well defined and ultraviolet finite in euclidean space-time. As a consequence the receipt of the Feynman integral calculation is the following [5, 7, 10, 11] : at first calculate the Feynman integrals in euclidean space-time and after make analytical continuation to Minkowski region. For nonlocal Feynman diagrams, analytical continuation to Minkowski region is straightforward, and it does not produce new singularities in comparison with local case. As it has been demonstrated [10,11] on the example of some Feynman diagrams, such prescription leads to well-defined results and preserves unitarity. It should be noted that analytical continuation to Minkowski region does not produce new singularities in comparison with the case of local field theory. Also it is possible to use the regularization formula (10) for the propagator for the calculation of Feynman integrals and after the calculations to perform the limit $\delta \rightarrow 0$. However from practical point of view, such calculations are extremely difficult.

整函数 $V\left(p^2\right)$ 在复无穷处存在本性奇点，导致维克转动的使用不成立。这意味着非局域费曼积分在闵可夫斯基时空 [5, 7, 10, 11] 中没有良好定义，而对应的积分在欧几里得时空中是定义良好且紫外有限的。因此，费曼积分的计算规则如下 [5, 7, 10, 11] : 首先在欧几里得时空中计算费曼积分，再解析延拓到闵可夫斯基区域。对于非局域费曼图，解析延拓到闵可夫斯基区域的过程是直接的，和局域情形相比不会产生新奇点。正如文献 [10,11] 在部分费曼图的例子中所证明的，这一规则能得到定义良好的结果，并且保持幺正性。需要注意，和局域场论的情形相比，解析延拓到闵可夫斯基区域不会产生新奇点。也可以在计算费曼积分时使用传播子的正则化公式 (10)，在计算完成后再取极限 $\delta \rightarrow 0$ 。但从实用角度来看，这类计算的难度极高。

Local Field Theory with Infinite Number of Local Fields as Origin of Nonlocality

以无穷多个局域场构成的局域场论作为非定域性的起源

In this section we discuss possible origin of nonlocality related with the introduction of infinite number of local scalar fields $\phi_n(x)$ [14, 15] . Consider the model with the Lagrangian:

本节我们讨论与引入无穷多个局域标量场 $\phi_n(x)$ [14, 15] 相关的非定域性的可能起源。考虑具有如下拉格朗日量的模型:

$$L_{\text{tot}} = L_0 + L_I \tag{37}$$

where

其中

$$L_o = \sum_{n=o}^{n=\infty} \frac{1}{2}\left(\partial^\mu \phi_n \partial_\mu \phi_n - M_n^2 \phi_n^2\right), \tag{38}$$

$$L_I = -g\phi_{eff}^4, \tag{39}$$

$$\phi_{\text{eff}}(x) = \sum_{n=0}^{n=\infty} c_n \left(-\partial^\mu \partial_\mu\right)^{n/2} \phi_n(x). \tag{40}$$

The propagator for the effective field $\phi_{eff}(x)$ is the infinite sum of local propagators, namely,

有效场 $\phi_{eff}(x)$ 的传播子是局域传播子的无穷求和，即

$$D_{eff}\left(p^2\right) = \sum_{n=0}^{n=\infty} c_n^2 \frac{\left(p^2\right)^n}{M_n^2 - p^2 - i\varepsilon}. \tag{41}$$

Imaginary part of the propagator (41) is nonnegative, and it coincides with the imaginary part of the propagator for the field $\phi_{eff}^0(x) = \sum\limits_{n=0}^{n=\infty} c_n (M_n)^n \phi_n(x)$. For the model (37-40) with $M_n^2 = M_1^2 + M_1^2 n$ and $c_n^2 = \frac{a^n}{n!}, a > 0$, the effective propagator (40) can be represented in the form:

式 (41) 传播子的虚部非负，且与场 $\phi_{eff}^0(x) = \sum\limits_{n=0}^{n=\infty} c_n (M_n)^n \phi_n(x)$ 传播子的虚部一致。对于包含 $M_n^2 = M_1^2 + M_1^2 n$ 和 $c_n^2 = \frac{a^n}{n!}, a > 0$ 的模型 (37-40)，有效传播子 (40) 可以表示为以下形式:

$$D_{eff}\left(p^2\right) = \frac{1}{M_1^2} \int_0^1 dx \exp\left(ap^2 x\right) x^{-p^2/M_1^2}. \tag{42}$$

In the euclidean region $q^2 = -p^2 > 0$ for $M_1^2 a > 1$, the propagator $D_{eff}\left(-q^2\right)$ has the asymptotics $D_{eff}\left(-q^2\right) \approx \frac{1}{M_1^2}\sqrt{\frac{2\pi}{q^2 M_1^2 a^2}} \exp\left[-\frac{q^2}{M_1^2} - \frac{q^2}{M_1^2}\ln\left(M_1^2 a\right)\right]$ at $q^2 \to \infty$. As a consequence all Feynman diagrams for the model (37-42) are ultraviolet finite. The model (37-42) describes an infinite number of local fields $\phi_n(x)$ with local interactions among them. An account of infinite number of local fields leads to nonlocal and ultraviolet finite theory. So we find that the introduction of infinite number of local fields could be an origin of nonlocality (For the model with finite number of scalar fields $\phi_n(x)\, n \le N_0$, the model (37-42) is nonrenormalizable. Only account of the infinite number of local fields $\phi_n(x)$ leads to ultraviolet finite theory.).

在欧几里得区域 $q^2 = -p^2 > 0$ 中，对于 $M_1^2 a > 1$ ，传播子 $D_{eff}\left(-q^2\right)$ 在 $q^2 \to \infty$ 处具有渐近行为 $D_{eff}\left(-q^2\right) \approx \frac{1}{M_1^2}\sqrt{\frac{2\pi}{q^2 M_1^2 a^2}} \exp\left[-\frac{q^2}{M_1^2} - \frac{q^2}{M_1^2}\ln\left(M_1^2 a\right)\right]$。因此，模型 (37-42) 的所有费曼图都是紫外有限的。模型 (37-42) 描述了无穷多个局域场 $\phi_n(x)$ ，它们之间存在局域相互作用。纳入无穷多个局域场会得到非定域且紫外有限的理论。因此我们发现，引入无穷多个局域场可以是非定域性的起源 (对于有限个标量场 $\phi_n(x)\, n \le N_0$ 的模型，模型 (37-42) 是不可重整的。只有纳入无穷多个局域场 $\phi_n(x)$ 才能得到紫外有限的理论。)。

Nonlocal Gauge Theories

非定域规范理论

Nonlocal QED

非局域量子电动力学 (QED)

As is well known the Lagrangian of QED

众所周知，QED 的拉格朗日量

$$L_{QED} = -\frac{1}{4}F^{\mu\upsilon}F_{\mu\upsilon} + \overline{\psi}\left(i\widehat{D} - m\right)\psi, \tag{43}$$

is invariant under gauge transformations

在规范变换下具有不变性

$$A_{\mu}(x) \to A_{\mu}(x) + \partial_{\mu}\alpha(x), \tag{44}$$

$$\psi(x) \to \exp(ie\alpha(x))\,\psi(x), \tag{45}$$

where $F_{\mu\nu} = \partial_{\nu}A_{\nu} - \partial_{\nu}A_{\mu}$ and $\widehat{D} = \gamma^{\mu}\partial_{\mu} - ie\gamma^{\mu}A_{\mu}$. It is possible to generalize local gauge transformations (44, 45) to nonlocal case, namely [6, 16] ,

其中 $F_{\mu\nu} = \partial_{\nu}A_{\nu} - \partial_{\nu}A_{\mu}$ 和 $\widehat{D} = \gamma^{\mu}\partial_{\mu} - ie\gamma^{\mu}A_{\mu}$。我们可以将局域规范变换 (44, 45) 推广到非局域情形，即 [6, 16] ,

$$A_{\mu}(x) \to A_{\mu}(x) + \partial_{\mu}\alpha(x), \tag{46}$$

$$\psi(x) \to \exp\left(ie\int d^4x' K(x-x')\,\alpha(x')\right)\psi(x). \tag{47}$$

Here $K(x)$ is some real function (formfactor) with the normalization condition:

此处 $K(x)$ 是满足以下归一化条件的某个实函数 (形状因子):

$$\int d^4x K(x) = 1 \tag{48}$$

The Lagrangian of nonlocal QED [6, 16]

非局域 QED 的拉格朗日量 [6, 16]

$$L_{nl,QED} = -\frac{1}{4}F^{\mu\upsilon}F_{\mu\upsilon} + \overline{\psi}\left(i\widehat{D}_{nl} - m\right)\psi \tag{49}$$

is invariant under nonlocal gauge transformations (46,47). Here

在非局域规范变换 (46,47) 下具有不变性。此处

$$\widehat{D}_{nl} = \gamma^{\mu}\partial_{\mu} - ie\int d^4x' K\left(x - x'\right)\gamma^{\mu}A_{\mu}\left(x'\right). \tag{50}$$

In nonlocal generalization of QED, we have to replace local interaction $e\overline{\psi}\gamma^{\mu}\psi A_{\mu}$ to nonlocal $e\overline{\psi}\gamma^{\mu}\psi A'_{\mu}$, where $A'_{\mu}\left(x\right) = \int d^4x' K\left(x - x'\right)A_{\mu}\left(x'\right)$. The Feynman rules for nonlocal QED coincide with the Feynman rules for local QED except the appearance of nonlocal formfactor $\widetilde{K}\left(k^2\right) = \int d^4x \exp\left(ikx\right)K\left(x\right)$ in the vertex, namely,

在 QED 的非局域推广中，我们需要将局域相互作用 $e\overline{\psi}\gamma^{\mu}\psi A_{\mu}$ 替换为非局域相互作用 $e\overline{\psi}\gamma^{\mu}\psi A'_{\mu}$，其中 $A'_{\mu}\left(x\right) = \int d^4x' K\left(x - x'\right)A_{\mu}\left(x'\right)$。非局域 QED 的费曼规则与局域 QED 的费曼规则一致，仅顶点处会多出非局域形状因子 $\widetilde{K}\left(k^2\right) = \int d^4x \exp\left(ikx\right)K\left(x\right)$，即:

$$\overline{\psi}\left(p + k\right)\gamma^{\mu}\psi\left(p\right)A_{\mu}\left(k\right) \to \widetilde{K}\left(k^2\right)\overline{\psi}\left(p + k\right)\gamma^{\mu}\psi\left(p\right)A_{\mu}\left(k\right). \tag{51}$$

For QED with several fermions ψ_i, the corresponding formfactors $\widetilde{K}_i\left(k^2\right)$ in general don’t coincide. For universal formfactor $\widetilde{K}_i\left(k^2\right) = \widetilde{K}\left(k^2\right)$ by the redefinition $A_{\mu}\left(x\right) \to A'_{\mu}\left(x\right) = \int d^4x' K\left(x - x'\right)A_{\mu}\left(x'\right)$, we can rewrite nonlocal Lagrangian (49) in the form:

对于含多个费米子 ψ_i 的 QED，对应的形状因子 $\widetilde{K}_i\left(k^2\right)$ 通常并不相同。对于通过重定义 $A_{\mu}\left(x\right) \to A'_{\mu}\left(x\right) = \int d^4x' K\left(x - x'\right)A_{\mu}\left(x'\right)$ 得到的普适形状因子 $\widetilde{K}_i\left(k^2\right) = \widetilde{K}\left(k^2\right)$，我们可以将非局域拉格朗日量 (49) 改写为如下形式:

$$L_{nl2,QED} = -\frac{1}{4}F^{\mu\upsilon}V^{-1}\left(-\partial^{\mu}\partial_{\mu}\right)F_{\mu\upsilon} + \overline{\psi}\left(i\widehat{D} - m\right)\psi, \tag{52}$$

where

其中

$$V\left(k^2\right) = \widetilde{K}^2\left(k^2\right). \tag{53}$$

In other words the simplest nonlocal generalization of QED consists in the replacement of local free photon Lagrangian to nonlocal one. For nonlocal QED (52), the Feynman rules coincide with standard QED Feynman rules except the replacement $\left(g_{\mu\nu} - \frac{k_{\mu}k_{\nu}}{k^2}\right)\frac{1}{k^2 + i\varepsilon} \to \left(g_{\mu\nu} - \frac{k_{\mu}k_{\nu}}{k^2}\right)\frac{V(k^2)}{k^2 + i\varepsilon}$ for local photon QED propagator in transverse gauge. For formfactor

换句话说，QED 最简单的非局域推广就是将局域自由光子拉格朗日量替换为非局域自由光子拉格朗日量。对于非局域 QED(52)，其费曼规则与标准 QED 的费曼规则一致，仅需要将横规范下的局域光子 QED 传播子替换为 $\left(g_{\mu\nu} - \frac{k_{\mu}k_{\nu}}{k^2}\right)\frac{1}{k^2 + i\varepsilon} \to \left(g_{\mu\nu} - \frac{k_{\mu}k_{\nu}}{k^2}\right)\frac{V(k^2)}{k^2 + i\varepsilon}$。对于形状因子

$$V\left(k^2\right)=O\left(\frac{1}{k^2}\right) \text{ at } k^2 \to -\infty \tag{54}$$

all Feynman diagrams except one-loop correction to the photon propagator are ultraviolet finite [6,16]. In nonlocal QED the interaction between two charges e_1 and e_2 is

除光子传播子的单圈修正外，所有费曼图都是紫外有限的 [6,16]。非局域 QED 中两个电荷 e_1 和 e_2 之间的相互作用为

$$W_{nl}\left(r\right)=\frac{e_1e_2}{\left(2\pi\right)^3}\int\frac{V\left(-\vec{k}^2\right)}{\vec{k}^2}\exp\left(-i\vec{k}\vec{x}\right)d\vec{k}. \tag{55}$$

For local QED with $V\left(-k^2\right)=1 W\left(r\right)=\frac{e_1e_2}{r}$ and $W\left(0\right)=\infty$, while for nonlocal QED with the propagator (54), the value $W_{nl}\left(r=0\right)$ is finite. It is possible to fix the form of nonlocal propagator $V\left(k^2\right)$ [6] by the requirement [6] that the interaction of two electrons in x -space $W_{nl}\left(r\right)$ is minimal at $r=0$. This requirement allows to fix the formfactor $V\left(z\right)$ equal to [6]

对于含 $V\left(-k^2\right)=1 W\left(r\right)=\frac{e_1e_2}{r}$ 和 $W\left(0\right)=\infty$ 的定域量子电动力学，其值发散；而对于带传播子 (54) 的非定域量子电动力学，值 $W_{nl}\left(r=0\right)$ 是有限的。我们可以通过以下要求确定非定域传播子 $V\left(k^2\right)$ [6] 的形式: 两个电子在 x 空间 $W_{nl}\left(r\right)$ 中的相互作用在 $r=0$ 处取极小值。该要求可确定形状因子 $V\left(z\right)$ 等于 [6]:

$$V\left(z\right)=K^2\left(z\right), \tag{56}$$

$$K\left(z\right)=\left[\sin\left(\left(1/2\right)lz^{1/2}\right)/\left(\left(1/2\right)lz^{1/2}\right)\right] \tag{57}$$

Here $l\equiv\frac{1}{\Lambda}$ is the inverse nonlocal scale. The formfactor (57) is an entire function of the minimal growth $\rho=1/2$, and it corresponds to the electron charge distribution [6]:

此处 $l\equiv\frac{1}{\Lambda}$ 是非定域特征尺度的倒数。式 (57) 的形状因子是极小增长 $\rho=1/2$ 的整函数，它对应电子的电荷分布 [6]:

$$b\left(\vec{r}^2\right)=\left(2\pi\right)^{-3}\int d\vec{k}K\left(\vec{k^2}\right)\exp\left(\vec{kr}\right)=\delta\left(r-l/2\right)/4\pi r^2, \tag{58}$$

The distribution (58) describes the uniformly charged sphere with the radius $l/2$.

分布 (58) 描述的是半径为 $l/2$ 的均匀带电球体。

Nonlocal Nonabelian Gauge Theories

非局域非阿贝尔规范理论

The straightforward generalization of nonlocal QED to nonabelian gauge theories consists in the replacement [9] of the local Yang-Mills Lagrangian:

将非局域量子电动力学直接推广到非阿贝尔规范理论，就是对局域杨-米斯拉格朗日量做替换 [9]:

$$L_{YM} = -\frac{1}{2}\operatorname{Tr}\left(F^{\mu\nu}F_{\mu\nu}\right) \to -\frac{1}{2}\operatorname{Tr}\left(F^{\mu\nu}V^{-1}\left(-\Delta^2\right)F_{\mu\nu}\right), \tag{59}$$

where $\Delta^2 = (\partial^\mu - igA^\mu)\left(\partial_\mu - igA_\mu\right), A_\mu = A^a_\mu T_a, \operatorname{Tr}\left(T_a T_b\right) = \frac{1}{2}\delta^a_b$. Nonlocal Lagrangian (59) is the generalization of Slavnov [17] regularization with higher-order derivatives. Slavnov regularization is gauge-invariant generalization of Pauli-Villars regularization to the case of nonabelian gauge fields. Slavnov regularization corresponds to the formfactor:

其中 $\Delta^2 = (\partial^\mu - igA^\mu)\left(\partial_\mu - igA_\mu\right), A_\mu = A^a_\mu T_a, \operatorname{Tr}\left(T_a T_b\right) = \frac{1}{2}\delta^a_b$ 。非局域拉格朗日量 (59) 是斯拉尔诺夫 [17] 高阶导数正则化的推广。斯拉尔诺夫正则化是将泡利-维拉尔正则化以规范不变的方式推广到非阿贝尔规范场的结果，对应形状因子为:

$$V^{-1}_{\text{Slavnov}}\left(-l^2\Delta^2\right) = 1 + c_k l^{2k}\left(\Delta^2\right)^k. \tag{60}$$

in the Lagrangian (59). The peculiarity of Slavnov regularization (60) is that it produces new vertices in the Lagrangian. For instance, for $k = 1$ the Lagrangian contains additional five and six vertices $V_5 \sim \partial AAAAA, V_6 \sim AAAAAA$. Slavnov has proved [17] that for $k \geq 2$, all diagrams are ultraviolet finite except some finite number of one-loop diagrams (For $k = 1$ all diagrams are ultraviolet finite except some one-loop and two-loop diagrams.). For instance, for $k = 2$, all diagrams are finite except one-loop propagators, three and four vertices. The increase of parameter k in Slavnov regularization (60) does not help to cure the remaining ultraviolet divergences. Note that in odd dimensions (d = 5 for instance), one-loop diagrams are well defined and ultraviolet finite in the sense of analytical continuation, and as a consequence Slavnov regularization for $k \geq 2$ leads to ultraviolet finite diagrams in all loops. From technical point of view, it is convenient to use nonlocal formfactor $V^{-1}\left(k^2\right)$ such that in the ultraviolet region $|k^2| \to \infty$

出现在拉格朗日量 (59) 中。斯拉尔诺夫正则化 (60) 的特殊之处在于它会在拉格朗日量中引入新顶点。例如，对 $k = 1$ ，拉格朗日量额外包含 5 点顶点和 6 点顶点 $V_5 \sim \partial AAAAA, V_6 \sim AAAAAA$ 。斯拉尔诺夫已证明 [17]: 对于 $k \geq 2$ ，除有限个单圈图外，所有费曼图都是紫外有限的 (对 $k = 1$ ，除部分单圈图和双圈图外，所有费曼图都是紫外有限的)。例如，对 $k = 2$ ，除单圈传播子、三点顶点和四点顶点外，所有费曼图都是有限的。增大斯拉尔诺夫正则化 (60) 中的参数 k 无法消除剩余的紫外发散。注意: 在奇数维 (例如 d=5) 中，单圈图是良定义的，且在解析延拓意义下是紫外有限的，因此对 $k \geq 2$ 使用斯拉尔诺夫正则化后，所有圈的图都是紫外有限的。从技术角度看，使用满足紫外区条件 $|k^2| \to \infty$ 的非局域形状因子 $V^{-1}\left(k^2\right)$ 会更方便

$$V^{-1}\left(k^2\right) \to p_N\left(k^2\right) = \sum_{n=0}^{n=N} c_n\left(k^2\right)^n. \tag{61}$$

The asymptotic formfactor $p_N\left(k^2\right)$ corresponds to local Lagrangian (59) with higher-order derivatives up to $2N + 2$. It means that in the ultraviolet region, the theory is asymptotically local and the corresponding behavior of Feynman diagrams in ultraviolet region is determined by the local Lagrangian $L_{as} = -\frac{1}{2}\operatorname{Tr}\left(F^{\mu\nu}p_N\left(-\Delta^2\right)F_{\mu\nu}\right)$. It should be noted that the conjecture that asymptotic behavior (61) of the formfactor $V^{-1}\left(k^2\right)$ determines the ultraviolet properties of the theory is not trivial. The reason is that subleading terms of the nonlocal formfactor $V^{-1}\left(-\Delta^2\right)$ generate additional vertices. At one-loop level using the Feynman rules for nonlocal gauge Lagrangian (59) (The Feynman rules for the nonlocal Lagrangian (59) for three and four vertices are contained in Ref. [18].), it is possible to check this conjecture. One can invent a lot of formfactors with the asymptotic behavior (61). For instance, nonlocal formfactors

渐近形状因子 $p_N\left(k^2\right)$ 对应最高到 $2N+2$ 阶导数的局域拉格朗日量 (59)。这意味着该理论在紫外区渐近局域，费曼图在紫外区的行为由局域拉格朗日量 $L_{as}=-\frac{1}{2}\operatorname{Tr}\left(F^{\mu\upsilon}p_N\left(-\Delta^2\right)F_{\mu\upsilon}\right)$ 决定。需要注意，形状因子 $V^{-1}\left(k^2\right)$ 的渐近行为 (61) 决定理论紫外性质这一猜想并非平凡结论，原因是非局域形状因子 $V^{-1}\left(-\Delta^2\right)$ 的次领头项会产生额外顶点。在单圈水平，可以利用非局域规范拉格朗日量 (59) 的费曼规则验证这一猜想 (非局域拉格朗日量 (59) 的三点顶点和四点顶点费曼规则可见参考文献 [18])。我们可以构造出大量满足渐近行为 (61) 的形状因子，例如如下非局域形状因子

$$V_{\sin}\left(k^2\right)=p_N^{-1}\left(k^2\right)\left[1-\frac{\sin\left(p_N^{1/2}\left(k^2\right)\right)}{p_N^{1/2}\left(k^2\right)}\right], \tag{62}$$

$$V_{\exp}\left(k^2\right)=p_N^{-1}\left(k^2\right)\left[1-\exp\left(-p_N\left(k^2\right)\right)\right], \tag{63}$$

are an entire function in k^2 complex plane and with ultraviolet asymptotics $p_N^{-1}\left(k^2\right)$. Other often used nonlocal formfactor of this type is [19]

是 k^2 复平面上的整函数，且满足紫外渐近行为 $p_N^{-1}\left(k^2\right)$ 。另一类常用的该型非局域形状因子见文献 [19]

$$V^{-1}(z)=\exp\left(\sum_{n=1}^{n=\infty}\frac{(-1)^{n+1}\left(p_N(z)\right)^{2n}}{2nn!}\right)=\exp\left(\frac{1}{2}\left[\Gamma\left(0,p_N(z)^2\right)+\gamma_E+\ln p_N(z)^2\right]\right),$$

(64)where $\Gamma(0,z)=\int_z^{+\infty}dte^{-t}/t$ is incomplete Gamma function with zero first argument and $\gamma_E=0.577$ is Euler constant. One can find that at $|z|\to\infty$

其中 $\Gamma(0,z)=\int_z^{+\infty}dte^{-t}/t$ 是第一自变量为零的不完全伽马函数，$\gamma_E=0.577$ 是欧拉常数。可以得到，当 $|z|\to\infty$

$$V^{-1}(z)\to V_\infty^{-1}=e^{\gamma_E/2}\left|p(z)\right|. \tag{65}$$

Nonlocal Gravity

非局域引力

The action of Einstein gravity without Λ -term and matter is (In this section we use flat metric $\eta_{\mu\nu}=\operatorname{Diag}(-+++)$.)

不含 Λ 项与物质的爱因斯坦引力作用量为 (本节我们采用平坦度规 $\eta_{\mu\nu}=\operatorname{Diag}(-+++)$ 。)

$$S[g] = \frac{1}{2\kappa^2}\int d^4xR\sqrt{-g}, \tag{66}$$

where

其中

$$R = g^{\mu\upsilon}R_{\mu\upsilon}, \tag{67}$$

$$R_{\mu\nu} = R^{\alpha}_{\mu\alpha\nu} = g^{\alpha\rho}R_{\alpha\mu\rho\nu}, \tag{68}$$

$$R^{\alpha}_{\mu\rho\upsilon} = \partial_\rho\Gamma^{\alpha}_{\mu\upsilon} - \partial_\upsilon\Gamma^{\alpha}_{\rho\mu} + \Gamma^{\alpha}_{\rho\beta}\Gamma^{\beta}_{\mu\upsilon} - \Gamma^{\alpha}_{\upsilon\beta}\Gamma^{\beta}_{\mu\rho}, \tag{69}$$

$$\Gamma^{\rho}_{\mu\nu} = \frac{1}{2}g^{\rho\lambda}\left(\partial_\mu g_{\lambda\nu} + \partial_\nu g_{\mu\lambda} - \partial_\lambda g_{\mu\nu}\right) \tag{70}$$

and $g = \mathrm{Det}\left(g_{\mu\upsilon}\right), \kappa^{-1} = \frac{M_{PL}}{\sqrt{8\pi}} = \left(2.4\cdot 10^{18}\mathrm{GeV}\right)$ is the reduced Planck mass. The use of perturbative expansion $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$ around flat space-time metric $\eta_{\mu\nu}$ leads to the following expansion on κ for the action of Einstein gravity:

且 $g = \mathrm{Det}\left(g_{\mu\upsilon}\right), \kappa^{-1} = \frac{M_{PL}}{\sqrt{8\pi}} = \left(2.4\cdot 10^{18}\mathrm{GeV}\right)$ 约化普朗克质量。围绕平坦时空度规 $\eta_{\mu\nu}$ 使用微扰展开 $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$ ，可得到爱因斯坦引力作用量关于 κ 的展开式如下:

$$S[g] = S_2 + \sum_{n=1}^{n=+\infty}\kappa^n S_{n+2} \tag{71}$$

Here S_2 is free graviton action, and S_{n+2} is the part of action describing self-interaction of $n+2$ gravitons. Free graviton propagator for the Einstein action (66) in transverse gauge $\partial^\mu h_{\mu\nu} = 0$ reads:

此处 S_2 是自由引力子作用量, S_{n+2} 是描述 $n+2$ 引力子自相互作用的作用量部分。横规范 $\partial^\mu h_{\mu\nu} = 0$ 下，爱因斯坦作用量 (66) 的自由引力子传播子为:

$$D_{GR,\mu\upsilon\rho\sigma}(k) = \frac{1}{k^2}\left(P^2_{\mu\upsilon\rho\sigma} - \frac{P^0_{\mu\upsilon\rho\sigma}}{2}\right), \tag{72}$$

where

其中

$$P^2_{\mu\upsilon\rho\sigma} = \frac{1}{2}\left(\theta_{\mu\rho}\theta_{\upsilon\sigma} + \theta_{\mu\sigma}\theta_{\upsilon\rho}\right) - \frac{1}{3}\theta_{\mu\upsilon}\theta_{\rho\sigma}, \tag{73}$$

$$P^0_{\mu\nu\rho\sigma} = \frac{1}{3}\theta_{\mu\nu}\theta_{\rho\sigma}, \tag{74}$$

$$\theta_{\mu\upsilon} = \eta_{\mu\upsilon} - \frac{k_\mu k_\upsilon}{k^2}. \tag{75}$$

Off-shell Einstein gravity (66) is nonrenormalizable at one-loop level [2]. On-shell Einstein gravity (66) is nonrenormalizable at two-loop level [3]. Using power counting rules and BRS invariance [20] K.S. Stelle proved [21] that the gravity action with four derivatives

脱壳爱因斯坦引力 (66) 在单圈层面不可重整 [2]，在壳爱因斯坦引力 (66) 在双圈层面不可重整 [3]。K.S. Stelle 利用幂次计数规则与 BRS 不变性 [20] 证明 [21]: 带四阶导数的引力作用量

$$S_{St}\,[g] = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\Big[R + \frac{1}{2}\left(\alpha R^2 + \beta R^{\mu\upsilon}R_{\mu\upsilon}\right)\Big], \tag{76}$$

is renormalizable field theory (Possible term $R_{\mu\upsilon\rho\sigma}R^{\mu\upsilon\rho\sigma}$ does not lead to new contribution to free graviton propagator due to the equality $R_{\mu\upsilon\rho\sigma}R^{\mu\upsilon\rho\sigma} - 4R^{\mu\upsilon}R_{\mu\upsilon} + R^2 = \mathrm{div}$, and we omit it. Here div stands for total covariant derivative.). It should be noted that the fact that the use of higher derivatives in kinetic term improves ultraviolet behavior and can make renormalizable nonrenormalizable theories is trivial and it is based on power-counting arguments. For instance, ϕ^4 model in $d = 8$ space-time becomes renormalizable for $\frac{1}{2}\phi\left(\partial^\mu\partial_\mu\right)^2\phi$ kinetic term in the Lagrangian. The price of such renormalizability is the existence of negative norm states in the spectrum. For the model (76), free graviton propagator has the form:

是可重整量子场论 (由于等式 $R_{\mu\upsilon\rho\sigma}R^{\mu\upsilon\rho\sigma} - 4R^{\mu\upsilon}R_{\mu\upsilon} + R^2 = \mathrm{div}$ ，可能存在的项 $R_{\mu\upsilon\rho\sigma}R^{\mu\upsilon\rho\sigma}$ 不会对自由引力子传播子产生新贡献，因此我们将其省略，此处 div 表示总协变导数。) 需要注意的是，动能项引入高阶导数可以改善紫外行为、让原本不可重整的理论变得可重整，这一结论是平凡的，它基于幂次计数论证。例如，拉格朗日中若 $\frac{1}{2}\phi\left(\partial^\mu\partial_\mu\right)^2\phi$ 动能项，ϕ^4 模型在 $d = 8$ 时空中就会变为可重整。这种可重整化的代价是谱中存在负范数态。对于模型 (76)，自由引力子传播子形式为:

$$\left(k^2\right)^{-1}\left[P^2_{\mu\nu\alpha\beta} - \frac{1}{2}P^0_{\mu\nu\alpha\beta}\right] - P^2_{\mu\nu\alpha\beta}\left(k^2 + m_2^2\right)^{-1} + \frac{1}{2}P^0_{\mu\nu\alpha\beta}\left(k^2 + m_0^2\right)^{-1}, \tag{77}$$

where $m_2 = (-\beta/2)^{-1/2}$ and $m_0 = (3\alpha + \beta)^{-1/2}$. The propagator (77) describes spin 2 massless graviton, spin 2 ghost with a mass m_2 , and spin zero scalar with a mass m_0 . The existence of ghost states in the spectrum makes impossible reasonable physical interpretation of the model.

其中 $m_2 = (-\beta/2)^{-1/2}$ 和 $m_0 = (3\alpha + \beta)^{-1/2}$ 。传播子 (77) 描述了自旋 2 质量为零的引力子、质量为 m_2 的自旋 2 鬼场，以及质量为 m_0 的自旋 0 标量场。谱中存在鬼态，导致该模型无法得到合理的物理解释。

A possible way to get rid of nonphysical ghost states and improve ultraviolet properties of the theory is the use of nonlocal gravity [15, 19, 22-25]. The simplest nonlocal gravity action has the form:

消除非物理鬼态、改善理论紫外性质的一种可行方案是采用非局域引力 [15, 19, 22-25]。最简单的非局域引力作用量形式为:

$$S_{nl}\,[g] = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\Big[R + \frac{1}{2}\left[Rf_1\left(\Box_{cov}\right)R + R^{\mu\upsilon}f_2\left(\Box_{cov}\right)R_{\mu\upsilon}\right]. \tag{78}$$

Here $\Box_{\mathrm{cov}} = g^{\mu\upsilon}\Delta_\mu\Delta_\upsilon$ is the covariant d' Alembertian $\left(\Box_{\mathrm{cov}}\,\phi = \frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}\ \left(\sqrt{-g}g^{\mu\upsilon}\frac{\partial\phi}{\partial x^\upsilon}\right)\right.$ for scalar field ϕ, $\Delta_l R_{ik} = \frac{\partial R_{ik}}{\partial x^l} - R_{mi}\Gamma^m_{kl} - R_{im}\Gamma^m_{kl}$, $\Delta_\mu A_\upsilon = \partial_\mu A_\upsilon - \Gamma^\alpha_{\mu\upsilon}A_\alpha)$ and $f_i = \sum f_{i,n}\left(\Box_{cov}\right)^n$ $(i = 1, 2)$ are nonlocal formfactors. The quadratic gravitational action for the action (78) reads [12,25]:

此处 $\square_{\text{cov}} = g^{\mu\upsilon}\Delta_\mu\Delta_\upsilon$ 是标量场 ϕ, $\Delta_l R_{ik} = \frac{\partial R_{ik}}{\partial x^l} - R_{mi}\Gamma^m_{kl} - R_{im}\Gamma^m_{kl}$, $\Delta_\mu A_\upsilon = \partial_\mu A_\upsilon - \Gamma^\alpha_{\mu\upsilon}A_\alpha$) 的协变达朗贝尔算符 $\left(\square_{\text{cov}}\phi = \frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}\left(\sqrt{-g}g^{\mu\upsilon}\frac{\partial\phi}{\partial x^\upsilon}\right)\right.$, $f_i = \sum f_{i,n}(\square_{cov})^n$ $(i = 1, 2)$ 是非定域形状因子。作用量 (78) 的二次引力作用量形式如下 [12,25]:

$$S_{(2)} = \frac{1}{4}\int d^4xL^{(2)}, \tag{79}$$

$$L^{(2)} = \frac{1}{2}h_{\mu\nu}f(\square)\square h^{\mu\nu} - h^\sigma_\mu f(\square)\partial_\sigma\partial_\nu h^{\mu\nu} + h^\mu_\mu g(\square)\partial_\mu\partial_\nu h^{\mu\nu} - \frac{1}{2}h^\mu_\mu g(\square)\square h^\mu_\mu$$

$$+\frac{1}{2}h^{\lambda\sigma}\frac{f(\square) - g(\square)}{\square}\partial_\lambda\partial_\sigma\partial_\mu\partial_\nu h^{\mu\nu}, \tag{80}$$

where

其中

$$f(\square) = 1 + \frac{1}{2}f_2(\square)\square, \tag{81}$$

$$g(\square) = 1 - 2f_1(\square)\square - \frac{1}{2}f_2(\square)\square. \tag{82}$$

For nonlocal action (78), gauge-independent part of free graviton propagator has the form:

对于非定域作用量 (78), 自由引力子传播子的规范无关部分形式为:

$$D_{\text{free},\mu\nu\rho\sigma}(k) = \frac{1}{f(-k^2)}D_{GR,\mu\nu\rho\sigma}(k) + \frac{3}{2}\frac{f(-k^2) - g(-k^2)}{(f(-k^2) - 3g(-k^2))k^2}P^0_{\mu\nu\rho\sigma},$$

(83)

where $D_{GR,\mu\upsilon\rho\sigma}(k)$ and $P^0_{\mu\upsilon\rho\sigma}$ are given by the formulae (72) and (74). For the case $f_2 = -2f_1$, only spin 2 propagates and the graviton propagator has the form:

其中 $D_{GR,\mu\upsilon\rho\sigma}(k)$ 和 $P^0_{\mu\upsilon\rho\sigma}$ 由式 (72) 和 (74) 给出。当 $f_2 = -2f_1$ 时，只有自旋 2 模式传播，引力子传播子形式为:

$$D_{\text{free},s=2,\mu\upsilon\rho\sigma}(k) = \frac{1}{f(-k^2)}D_{GR,\mu\upsilon\rho\sigma}(k), \tag{84}$$

$$f(-k^2) = 1 - \frac{1}{2}f_2(-k^2)k^2. \tag{85}$$

We assume that the formfactor $f^{-1}(-k^2)$ is an entire function of the growth $\rho \geq 1/2$. As a consequence of this assumption, free graviton propagator (84) does not have singularities in complex k^2 plane except graviton pole at $k^2 = 0$. So we can expect that the theory is unitary, and moreover for the decreasing formfactor $f^{-1}(-k^2) \sim O\left(\left(\frac{1}{k^2}\right)^2\right)$ in euclidean region at $k^2 \to +\infty$, the model is superrenormalizable, and technical details are contained in review [25]. Especially interesting are the formfactors (85) with the asymptotics $f_2(-k^2) \to P_N(k^2)$ at $k^2 \to +\infty$. The ultraviolet asymptotics of such formfactors coincides with local formfactor $f_{2,loc}(k^2) = P_N(k^2)$ that corresponds to the local gravity model with finite number of higher-order derivatives. In fact such models are the generalizations of Slavnov regularization with higher-order derivatives to the gravity case. Using power counting of ultraviolet divergences and BRS invariance [20] of the gravity action, one can find that the models with polynomial formfactors are superrenormalizable in full analogy with Slavnov higher-order regularization for nonabelian gauge theories (See also P. Lavrov, I. Shapiro, and N. Ohta this handbook.). However the spectrum of such models contains negative norm states that makes very difficult or even impossible reasonable physical interpretation. The use of nonlocal formfactor $f\left(-k^2\right)$ with polynomial behavior at ultraviolet asymptotics leads to superrenormalizable theory and allows to get rid of problems with the presence in the spectrum of negative norm states; see as a review [25]. It should be stressed that up to now, there is no rigorous proof that subleading terms in the nonlocal formfactor don't generate new ultraviolet divergences. The reason is that subleading terms in formfactor change not only propagator but vertices.

我们假设形状因子 $f^{-1}\left(-k^2\right)$ 是增长阶为 $\rho \geq 1/2$ 的整函数。该假设下，自由引力子传播子 (84) 在复 k^2 平面上除 $k^2=0$ 处的引力子极点外没有其他奇点，因此可以预期该理论是幺正的；此外，对于欧氏区域 $k^2 \to +\infty$ 处递减的形状因子 $f^{-1}\left(-k^2\right) \sim O\left(\left(\frac{1}{k^2}\right)^2\right)$，该模型是超可重整化的，技术细节参见综述文献 [25]。尤其值得关注的是在 $k^2 \to +\infty$ 处具有渐近行为 $f_2\left(-k^2\right) \to P_N\left(k^2\right)$ 的形状因子 (85)。这类形状因子的紫外渐近行为与定域形状因子 $f_{2,loc}\left(k^2\right) = P_N\left(k^2\right)$ 一致，后者对应有限阶高阶导数定域引力模型。实际上这类模型是斯拉沃诺夫高阶导数正则化向引力情形的推广。通过对紫外发散做幂次计数并利用引力作用量的 BRS 不变性 [20]，可以发现多项式形状因子模型都是超可重整化的，完全类似非阿贝尔规范理论中的斯拉沃诺夫高阶正则化 (另见本手册中 P. Lavrov, I. Shapiro, N. Ohta 的内容)。但这类模型的谱中存在负范态，导致很难甚至不可能给出合理的物理解释。采用紫外渐近为多项式行为的非定域形状因子 $f\left(-k^2\right)$ 可以得到超可重整化理论，同时解决谱中存在负范态的问题，综述参见 [25]。需要强调的是，到目前为止尚无严格证明证明非定域形状因子中的次领头项不会产生新的紫外发散，原因在于形状因子中的次领头项不仅会改变传播子，还会改变顶点。

As it was mentioned before in nonlocal QED, the interaction between the charges could be finite at $r=0$ for decreasing at $k^2 \to -\infty$ as $\left(k^2\right)^{-1}$ nonlocal formfactor. Absolutely the same situation takes place for nonlocal gravity. Classical aspects of nonlocal gravity are discussed in [12] and A. Mazumdar this handbook.

正如之前在非局域量子电动力学中提到的，对于在 $k^2 \to -\infty$ 处递减的 $\left(k^2\right)^{-1}$ 非局域形状因子，电荷间的相互作用在 $r=0$ 处可以是有限的。非局域引力的情况与此完全相同。非局域引力的经典性质已在文献 [12] 以及本手册 A. Mazumdar 的文章中讨论。

A very important question naturally arises: what about the scale of nonlocality Λ ?. It is clear that nonlocal scale Λ has to be smaller or equal to the scale κ^{-1} because in opposite case we shall have the problems with tree-level unitarity for graviton amplitudes. The most natural assumption is that $\Lambda \sim O\left(\kappa^{-1}\right)$, but we can't exclude the case $\Lambda \ll \kappa^{-1}$. Current experimental data support Starobinsky R^2 model of the inflation. There are attempts [26]; see also A. Koshelev, S. Kumar, and A. Starobinsky this handbook to use nonlocal gravity in the inflation models. It is interesting to mention that in Starobinsky model [27, 28] , the free parameter is the scalar mass $M \approx 3 \cdot 10^{13}$GeV , and the nonlocal scale Λ has to be much larger the scalar mass $\Lambda \gg M$. So Starobinsky model gives us the hint that nonlocal scale $\Lambda \geq 10^{14}$GeV .

一个非常重要的问题自然会被提出: 非局域性的标度 Λ 是多少? 显然非局域标度 Λ 必须小于或等于标度 κ^{-1}，否则我们的引力子振幅树级别幺正性会出现问题。最自然的假设是 $\Lambda \sim O\left(\kappa^{-1}\right)$，但我们也不能排除 $\Lambda \ll \kappa^{-1}$ 的情况。现有实验数据支持 Starobinsky R^2 暴涨模型。已有研究 [26] 尝试将非局域引力用于暴涨模型；也可参见本手册中 A. Koshelev、S. Kumar 与 A. Starobinsky 的相关文章。值得一提的是，在 Starobinsky 模型 [27, 28] 中，自由参数是标量质量 $M \approx 3 \cdot 10^{13}$GeV，且非局域标度 Λ 必须远大于该标量质量 $\Lambda \gg M$。因此 Starobinsky 模型给了我们提示: 非局域标度 $\Lambda \geq 10^{14}$GeV。

Renormalizable Nonlocal Gravity

可重整化非局域引力

It is well known that the SM- renormalizable gauge theory based on $SU_c(3) \otimes\ SU_L(2) \otimes U(1)$ gauge theory successfully describes strong, electromagnetic, and weak interactions. So it is natural to suggest that quantum gravity also could be renormalizable field theory. However Stelle renormalizable local generalization (76) of Einstein gravity (66) contains indefinite metric spin 2 state in graviton propagator. The formfactor $f\left(-k^2\right)$ in graviton propagator (84) with ultraviolet asymptotics $f\left(-k^2\right)/k^2 \to$ const at $k^2 \to \infty$ corresponds to asymptotically renormalizable theory (76), and besides it provides the absence of negative norm states at least for free graviton propagator. For instance, we can use nonlocal formfactor $f^{-1}\left(-k^2\right) = \Lambda^2\left(\frac{\left(1-\exp\left(-\frac{k^2}{\Lambda^2}\right)\right)}{k^2}\right)$ in formula (85) or the formfactors (56,64). The theory with such formfactors is renormalizable in the ultraviolet asymptotics, and graviton propagator (84) does not contain negative norm states. The main problem is that in contrast to local field theories, nonlocal renormalizable gravity depends on unknown formfactor $f\left(k^2\right)$, and up to now there are no fundamental principles how to fix its form.

众所周知，基于 $SU_c(3) \otimes\ SU_L(2) \otimes U(1)$ 规范理论的标准模型可重整化规范理论成功描述了强相互作用、电磁相互作用和弱相互作用。因此自然可以提出，量子引力也可以是可重整化场论。然而，爱因斯坦引力 (66) 的施泰勒可重整化局域推广 (76) 的引力子传播子中存在不定度规自旋 2 态。引力子传播子 (84) 中的形状因子 $f\left(-k^2\right)$ 在 $k^2 \to \infty$ 处具有紫外渐近性 $f\left(-k^2\right)/k^2 \to$ 常数，对应渐近可重整化理论 (76)，此外它保证了至少自由引力子传播子中不存在负范数态。例如，我们可以在式 (85) 中使用非局域形状因子 $f^{-1}\left(-k^2\right) = \Lambda^2\left(\frac{\left(1-\exp\left(-\frac{k^2}{\Lambda^2}\right)\right)}{k^2}\right)$，或使用形状因子 (56,64)。含这类形状因子的理论在紫外渐近区是可重整化的，且引力子传播子 (84) 不包含负范数态。核心问题在于，和局域场论不同，非局域可重整化引力依赖于未知形状因子 $f\left(k^2\right)$，迄今为止尚无确定其形式的基本原理。

Some Applications of Nonlocal Field Theory

非局域场论的若干应用

γ_5 -Anomaly and Nonlocal Theories

γ_5 反常与非定域理论

It is well known that triangle γ_5 -anomalies spoil the gauge invariance at one-loop level [29, 30] . As a consequence longitudinal and transverse photons interact at one-loop level that makes impossible physical interpretation of γ_5 -anomalous models. As a simplest example, consider axial QED with the Lagrangian:

众所周知，三角 γ_5 反常会在单圈阶破坏规范不变性 [29, 30] 。由此，纵光子与横光子会在单圈阶发生相互作用，导致 γ_5 反常模型无法给出物理解释。以轴矢量子电动力学为例，其拉氏量为:

$$L_A = -\frac{1}{4}F^{\mu\upsilon}F_{\mu\upsilon} + i\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi + e\overline{\psi}\gamma^{\mu}\gamma_5 A_{\mu}\psi. \tag{86}$$

At the classical level, the Lagrangian (86) is invariant under gauge transformations:

经典层面，拉氏量 (86) 在如下规范变换下保持不变:

$$\psi \to \exp\left(ie\gamma_5\alpha\right)\psi,\ A_{\mu} \to A_{\mu} + \partial_{\mu}\alpha. \tag{87}$$

Due to γ_5 -anomaly, the effective Lagrangian

由于 γ_5 反常，有效拉氏量

$$L_{eff}\left(A_{\mu}\right) = \frac{1}{i}\ln\left(\int d\psi d\overline{\psi}\exp\left(i\int d^4xL_A\right)\right) \tag{88}$$

is not invariant under gauge transformations, namely [29, 30] ,

在规范变换下不再不变，即对 [29, 30] ,

$$L_{eff}\left(A_{\mu} + \partial_{\mu}\alpha\right) - L_{eff}\left(A_{\mu}\right) = \frac{\alpha e^3}{12\pi^2}\varepsilon^{\mu\upsilon\alpha\beta}F_{\mu\upsilon}F_{\alpha\beta}. \tag{89}$$

To restore the gauge invariance (87) at quantum level, let us add additional scalar field ϕ which transforms as $\phi \to \phi + \alpha$ under the gauge transformations (87) and add the term [31]:

为在量子层面恢复规范不变性 (87)，我们引入额外的标量场 ϕ ，它在规范变换 (87) 下的变换形式为 $\phi \to \phi + \alpha$ ，并添加如下项 [31]:

$$L_A \to L_{\text{ tot }} = L_A + \Delta L, \tag{90}$$

$$\Delta L = \frac{1}{2}m_A^2\left(A_{\mu} - \partial_{\mu}\phi\right)\left(A^{\mu} - \partial^{\mu}\phi\right) - \phi\frac{e^3}{12\pi^2}\varepsilon^{\mu\upsilon\alpha\beta}F_{\mu\upsilon}F_{\alpha\beta}. \tag{91}$$

The Lagrangian ΔL restores the gauge invariance at quantum level. Due to the presence of the term $-\phi\frac{e^3}{12\pi^2}\varepsilon^{\mu\upsilon\alpha\beta}F_{\mu\upsilon}F_{\alpha\beta}$ in (91), the Lagrangian $L + \Delta L$ is nonrenormalizable.

拉氏量 ΔL 可以在量子层面恢复规范不变性。由于 (91) 中存在项 $-\phi\frac{e^3}{12\pi^2}\varepsilon^{\mu\upsilon\alpha\beta}F_{\mu\upsilon}F_{\alpha\beta}$，拉氏量 $L+\Delta L$ 是不可重整的。

So we see that by the introduction of additional scalar field ϕ, it is possible to restore the gauge invariance at quantum level. However the price of the gauge invariance restoration is the lack of the renormalizability. The nonlocal analog of the Lagrangian $L_A+\Delta L$ allows in full analogy with nonlocal QED to make the model (86) superrenormalizable [32-34]. Namely, consider the nonlocal generalization of the model (90) with the Lagrangian:

因此我们看到，引入额外标量场 ϕ 可以在量子层面恢复规范不变性，但恢复规范不变性的代价是理论失去可重整性。拉氏量 $L_A+\Delta L$ 的非定域类比和非定域量子电动力学完全类似，可以让模型 (86) 变成超可重整的 [32-34]，即我们考虑模型 (90) 的非定域推广，其拉氏量为:

$$L_{tot,nl}=L_{A,nl}+\Delta_{nl}L, \tag{92}$$

$$L_{A,nl}=-\frac{1}{4}F^{\mu\upsilon}V_{tr}^{-1}\left(-\Box\right)F_{\mu\upsilon}+i\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi+e\overline{\psi}\gamma^{\mu}\gamma_5A_{\mu}\psi, \tag{93}$$

$$\Delta_{nl}L=\frac{1}{2}m_A^2\left(A_{\mu}-\partial_{\mu}\phi\right)V_l^{-1}\left(-\Box\right)\left(A^{\mu}-\partial^{\mu}\phi\right)-\phi\frac{e^3}{12\pi^2}\varepsilon^{\mu\upsilon\alpha\beta}F_{\mu\upsilon}F_{\alpha\beta}. \tag{94}$$

For the Lagrangian (92) the vector propagator in transverse gauge is

对于拉氏量 (92)，横规范下的矢量传播子为

$$D_{AA}^{\mu\upsilon}\left(k\right)=\frac{\left(g^{\mu\upsilon}-\frac{k^{\mu}k^{\upsilon}}{k^2}\right)V_{tr}\left(k^2\right)}{k^2-m_A^2V_{tr}\left(k^2\right)V_l^{-1}\left(k^2\right)} \tag{95}$$

while scalar propagator is

而标量传播子为

$$D_{\phi\phi}=\frac{-V_l\left(k^2\right)}{k^2m_A^2}. \tag{96}$$

Consider at first the case $V_l\left(k^2\right)=1$. Using power-counting arguments, one can show that in full analogy with nonlocal QED for the transverse formfactor $V_{tr}\left(k^2\right)$ with the ultraviolet asymptotics $V_{tr}\left(k^2\right)=O\left(\frac{1}{\left(k^2\right)^2}\right)$, all Feynman diagrams except one-loop correction to the vector propagator are ultraviolet finite. For the case $V_{tr}\left(k^2\right)=V_l\left(k^2\right)$, one can show that for the longitudinal formfactor with the ultraviolet asymptotics $V_{tr}\left(k^2\right)=O\left(\frac{1}{\left(k^2\right)^2}\right)$ all diagrams except one-loop correction to the vector propagator are ultraviolet finite. In the unitary gauge, the most general form of vector propagator is

我们首先考虑 $V_l(k^2)=1$ 的情况。通过量纲计数可以证明，和非定域量子电动力学完全类似，对于具有紫外渐近行为 $V_{tr}(k^2)=O\left(\frac{1}{(k^2)^2}\right)$ 的横形状因子 $V_{tr}(k^2)$，除矢量传播子的单圈修正外，所有费曼图都是紫外有限的。对于 $V_{tr}(k^2)=V_l(k^2)$ 的情况，可以证明，对于具有相同紫外渐近行为 $V_{tr}(k^2)=O\left(\frac{1}{(k^2)^2}\right)$ 的纵形状因子，除矢量传播子的单圈修正外，所有费曼图也都是紫外有限的。幺正规范下，矢量传播子的最一般形式为

$$D^{\mu\upsilon}_{AA,u}(k)=\frac{\left(g^{\mu\upsilon}V_1(k^2)-\frac{k^\mu k^\upsilon}{m_A^2}V_2(k^2)\right)}{k^2-m_A^2}. \tag{97}$$

The longitudinal formfactor $V_2(k^2)$ does not contribute at mass shell for γ_5 nonanomalous models like QED. For $V_1(k^2)=\frac{k^2}{m_A^2}V_2(k^2)$, the vector propagator (97) is transverse, and triangle γ_5 -anomaly does not contribute.

对于像量子电动力学这样的 γ_5 无反常模型，纵形状因子 $V_2(k^2)$ 在质壳上没有贡献。对于 $V_1(k^2)=\frac{k^2}{m_A^2}V_2(k^2)$，矢量传播子 (97) 是横的，因此三角 γ_5 反常没有贡献。

To conclude, in axial QED it is possible to restore gauge invariance by the introduction of additional scalar field. However due to γ_5 -anomalies, the local version of the model with $V_{tr}(k^2)=V_l(k^2)=1$ is non-renormalizable. The introduction of nonlocal formfactors makes the model superrenormalizable. It should be stressed that in nonlocal axial QED, it is impossible to perform the limit $m_A\to 0$, i.e., axial QED with massless vector field does not exist.

综上所述，在轴矢量子电动力学中，可以通过引入额外标量场来恢复规范不变性。但由于 γ_5 反常，包含 $V_{tr}(k^2)=V_l(k^2)=1$ 的定域版本模型是不可重整化的。引入非定域形状因子后，该模型变为超可重整化模型。需要强调的是，在非定域轴矢量子电动力学中，无法取极限 $m_A\to 0$，也就是说，无质量矢量场的轴矢量子电动力学并不存在。

Nonlocal SU(5) GUT

非定域 SU(5) 大统一理论

The remarkable success of the supersymmetric $SU(5)$ grand unified theory (GUT) (As a review see [35].) was considered by many physicists as the first hint in favor of the existence of low energy broken supersymmetry in nature. However the nonobservation of supersymmetry at the LHC is probably the opposite hint that the supersymmetry concept and in particular the supersymmetric $SU(5)$ GUT are wrong.

超对称 $SU(5)$ 大统一理论 (GUT) 的非凡成功 (综述见 [35]) 被许多物理学家视为自然界存在低能破缺超对称的首个佐证。然而大型强对撞机尚未观测到超对称，这一事实恰恰反过来表明，超对称概念、尤其是超对称 $SU(5)$ GUT 很可能是错误的。

It is well known that the Georgi-Glashow $SU(5)$ GUT [36] is in conflict with experimental data [37]. So a natural question arises: is it possible to invent nonsupersymmetric generalizations of the standard $SU(5)$ GUT noncontradicting to the experimental data? The answer is positive, in particular, the introduction of additional split multiplets $5 \oplus \overline{5}$ and $10 \oplus \overline{10}$ in the Georgi-Glashow $SU(5)$ GUT allows to obtain the Weinberg angle θ_w in agreement with experiment [38].

众所周知，乔治-格拉肖 $SU(5)$ GUT[36] 与实验数据 [37] 相矛盾。因此一个自然的问题应运而生: 能否构造出与实验数据不矛盾的标准 $SU(5)$ GUT 的非超对称推广？答案是肯定的: 具体而言，在乔治-格拉肖 $SU(5)$ GUT 中引入额外的分裂多重态 $5 \oplus \overline{5}$ 和 $10 \oplus \overline{10}$，就可以得到与实验 [38] 一致的温伯格角 θ_w 。

In this section we discuss nonlocal generalization of $SU(5)$ GUT [39]. We demonstrate that it is possible to solve the problems with the proton lifetime and the Weinberg angle by the introduction of additional nonlocal terms in the local $SU(5)$ GUT Lagrangian. An account of additional terms leads to the modification of the GUT condition $\alpha_1(M_{GUT}) = \alpha_2(M_{GUT}) = \alpha_3(M_{GUT})$ for the effective coupling constants. Nonlocal scale Λ responsible for ultraviolet cutoff coincides (up to some factor) with GUT scale M_{GUT} . In the simplest nonlocal modification of the standard renormalizable $SU(5)$ GUT, the value of the GUT scale is $M_{GUT} \approx 3 \cdot 10^{16}\mathrm{GeV}$. In general case the value of M_{GUT} is an arbitrary, and the most interesting option $M_{GUT} = O(M_{PL})$ could be realized.

本节我们讨论 $SU(5)$ GUT 的非定域推广 [39]。我们证明，通过在定域 $SU(5)$ GUT 拉格朗日量中引入额外非定域项，可以解决质子寿命和温伯格角的问题。额外项的引入修正了有效耦合常数满足的大统一条件 $\alpha_1(M_{GUT}) = \alpha_2(M_{GUT}) = \alpha_3(M_{GUT})$ 。负责紫外截断的非定域标度 Λ (相差一个常数因子) 与大统一标度 M_{GUT} 一致。在标准可重整 $SU(5)$ GUT 的最简非定域修正中，大统一标度的取值为 $M_{GUT} \approx 3 \cdot 10^{16}\mathrm{GeV}$ 。一般情况下 M_{GUT} 的取值是任意的，最有意思的情形 $M_{GUT} = O(M_{PL})$ 可以实现。

Let us start with the observation that in the SM, the effective coupling constants $\alpha_3(\mu)$ and $\alpha_2(\mu)$ cross each other ($\alpha_3(M_{GUT}) = \alpha_2(M_{GUT})$) at the scale $M_{GUT} \approx O\left(10^{17}\right)\mathrm{GeV}$. At one-loop level the effective coupling constants $\alpha_i(\mu)$ obey the equations:

我们首先来看标准模型中的结果: 有效耦合常数 $\alpha_3(\mu)$ 和 $\alpha_2(\mu)$ 在标度 $M_{GUT} \approx O\left(10^{17}\right)\mathrm{GeV}$ 处相交 ($\alpha_3(M_{GUT}) = \alpha_2(M_{GUT})$)。单圈水平下，有效耦合常数 $\alpha_i(\mu)$ 满足以下方程:

$$\mu \frac{d\alpha_i(\mu)}{d\mu} = \frac{b_i}{2\pi} \alpha_i^2(\mu), \tag{98}$$

where for the SM model with three generations $b_3 = -7, b_2 = -3\frac{1}{6}$, and $b_1 = 4.1$. As a consequence we find that

其中对于三代标准模型，有 $b_3 = -7, b_2 = -3\frac{1}{6}$ 和 $b_1 = 4.1$ 。由此我们得到:

$$\frac{1}{\alpha_2(m_t)} - \frac{1}{\alpha_3(m_t)} = \frac{b_2 - b_3}{2\pi} \ln\left(\frac{M_{GUT}}{m_t}\right). \tag{99}$$

Numerically $M_{GUT} = (0.9 \pm 0.2) \cdot 10^{17}\mathrm{GeV}$ and $\frac{1}{\alpha_3(M_{GUT})} = 46.9 \pm 0.2$ (In our estimates we use $\alpha_3(m_Z) = 0.118 \pm 0.001, \sin^2(\theta_W)(m_Z) = 0.231 \pm 0.001$, and $\alpha_{em}^{-1}(m_Z) = 127.8 \pm 0.1$.).

数值结果为 $M_{GUT}=(0.9\pm 0.2)\cdot 10^{17}\mathrm{GeV}$ 和 $\frac{1}{\alpha_3(M_{GUT})}=46.9\pm 0.2$ (我们的估计中使用了 $\alpha_3(m_Z)=0.118\pm 0.001, \sin^2(\theta_W)(m_Z)=0.231\pm 0.001$ 和 $\alpha_{em}^{-1}(m_Z)=127.8\pm 0.1$)。

The unification scale $M_{GUT}=(0.9\pm 0.2)\cdot 10^{17}\mathrm{GeV}$ is safe for the current proton decay bound [40]. Really, in standard $SU(5)$ model, the proton lifetime due to the massive vector exchange is determined by the formula [41]:

大统一能标 $M_{GUT}=(0.9\pm 0.2)\cdot 10^{17}\mathrm{GeV}$ 满足当前质子衰变界限的要求 [40]。实际上, 在标准 $SU(5)$ 模型中，重矢量交换导致的质子寿命由下式给出 [41]:

$$\Gamma(p\to e^{+}\pi^{o})^{-1}=4\cdot 10^{29\pm 0.7}\left(\frac{M_{v}}{2\cdot 10^{14}\mathrm{GeV}}\right)^{4}\mathrm{yr}, \tag{100}$$

where $M_v\equiv M_{GUT}=\sqrt{\frac{5}{24}}g_5\Phi_0$ is the mass of vector bosons responsible for proton decay (Here Φ_0 is the vacuum expectation value of the $SU(5)$ scalar 24-plet $<\Phi>=\frac{\Phi_0}{\sqrt{15}}\mathrm{Diag}(1,1,1,-3/2,-3/2)$ responsible for $SU(5)\to SU_c(3)\otimes SU_L(2)\otimes U(1)$ gauge symmetry breaking and g_5 is the $SU(5)$ gauge coupling at the GUT scale M_{GUT} .). From the current experimental limit [40] $\Gamma(p\to e^{+}\pi^{o})^{-1}\geq 1.67\cdot 10^{34}\mathrm{yr}$, we conclude that $M_{GUT}\geq 2.5\cdot 10^{15}\mathrm{GeV}$. The main problem of the standard $SU(5)$ GUT with the unification scale $M_{GUT}\approx 10^{17}\mathrm{GeV}$ is that the experimental values of $\alpha_3(m_Z), \sin^2(\theta_W)(m_Z)$, and $\alpha_{em}^{-1}(m_Z)$ lead to nonequal values of the effective coupling constants $\alpha_3(M_{GUT})$ and $\alpha_1(M_{GUT})$, namely, $\alpha_1^{-1}(M_{GUT})=36.0\neq\alpha_3^{-1}(M_{GUT})=46.9$.

其中 $M_v\equiv M_{GUT}=\sqrt{\frac{5}{24}}g_5\Phi_0$ 是引发质子衰变的矢量玻色子质量 (此处 Φ_0 是负责 $SU(5)\to SU_c(3)\otimes SU_L(2)\otimes U(1)$ 规范对称性破缺的 $SU(5)$ 标量 24 重态 $<\Phi>=\frac{\Phi_0}{\sqrt{15}}\mathrm{Diag}(1,1,1,-3/2,-3/2)$ 的真空期望值，g_5 是大统一能标 M_{GUT} 处的 $SU(5)$ 规范耦合常数。) 根据当前实验上限 [40] $\Gamma(p\to e^{+}\pi^{o})^{-1}\geq 1.67\cdot 10^{34}\mathrm{yr}$ ，我们得到结论 $M_{GUT}\geq 2.5\cdot 10^{15}\mathrm{GeV}$ 。标准 $SU(5)$ 大统一理论中大统一能标取 $M_{GUT}\approx 10^{17}\mathrm{GeV}$ 的核心问题是，实验测得的 $\alpha_3(m_Z), \sin^2(\theta_W)(m_Z)$ 和 $\alpha_{em}^{-1}(m_Z)$ 给出的有效耦合常数 $\alpha_3(M_{GUT})$ 与 $\alpha_1(M_{GUT})$ 不相等，即 $\alpha_1^{-1}(M_{GUT})=36.0\neq\alpha_3^{-1}(M_{GUT})=46.9$ 。

The main observation [39] is that the use of nonrenormalizable interaction (In Refs. [42,43] the influence of nonrenormalizable interaction $L_{nl}=\frac{c}{M_{PL}}\mathrm{Tr}\left(F_{\mu\nu}\Phi F^{\mu\nu}\right)$ with $c=O(1)$ has been studied. It was realized that this interaction allows to increase the GUT scale but can't solve the problem with wrong Weinberg angle prediction.)

核心观测结论 [39] 是，使用不可重整相互作用 (文献 [42,43] 已经研究了不可重整相互作用 $L_{nl}=\frac{c}{M_{PL}}\mathrm{Tr}\left(F_{\mu\nu}\Phi F^{\mu\nu}\right)$ 与 $c=O(1)$ 的影响，发现该相互作用可以提高大统一能标，但无法解决温伯格角预测错误的问题。)

$$\Delta L_{F\Phi F\Phi}=\frac{1}{4\Lambda_{\Phi 1}^{2}}\left(\mathrm{Tr}\left(F_{\mu\nu}\Phi\right)\mathrm{Tr}\left(F^{\mu\nu}\Phi\right)\right) \tag{101}$$

leads to additional term for the effective coupling constant $\alpha_1(\mu)$ at GUT scale, namely,

会给大统一能标处的有效耦合常数 $\alpha_1(\mu)$ 带来额外项，即

$$\frac{1}{\alpha_1\left(M_{GUT}\right)}=\frac{1}{\alpha_3\left(M_{GUT}\right)}-\Delta, \tag{102}$$

where

其中

$$\Delta=\frac{\pi\Phi_o^2}{\Lambda_{\Phi 1}^2}=\frac{1}{\alpha_3\left(M_{GUT}\right)}\frac{6M_v^2}{5\Lambda_{\Phi 1}^2}. \tag{103}$$

Numerically we obtain $\Delta=10.9\pm0.2$ and $\Lambda_{\Phi 1}\approx 2.3\cdot M_v$. In other words additional nonrenormalizable interaction (101) can modify GUT unification condition in such a way that the unification takes place at the GUT scale nondangerous for proton decay bound and the GUT scale $M_{GUT}\approx 10^{17}$GeV does not contradict to the experimental values of $\sin^2\left(\theta_W\right)\left(M_Z\right)$ and $\alpha^{-1}\left(M_Z\right)$. The appearance of additional arbitrary parameter Δ in the relation (102) means that we can' t predict the value of $\sin^2\left(\theta_W\right)$. The untrivial fact is that the unifcation of $\alpha_2\left(\mu\right)$ and $\alpha_3\left(\mu\right)$ effective coupling constants takes place at the GUT scale which is safe for the proton lifetime bound.

通过数值计算我们得到 $\Delta=10.9\pm0.2$ 和 $\Lambda_{\Phi 1}\approx 2.3\cdot M_v$ 。换言之，额外的不可重整相互作用 (101) 可以对大统一理论的统一条件做出如下修改: 统一过程发生在对质子衰变界无害的大统一能标处，且大统一能标 $M_{GUT}\approx 10^{17}$GeV 与 $\sin^2\left(\theta_W\right)\left(M_Z\right)$ 和 $\alpha^{-1}\left(M_Z\right)$ 的实验值不矛盾。关系式 (102) 中出现额外任意参数 Δ ，意味着我们无法预测 $\sin^2\left(\theta_W\right)$ 的值。值得注意的非平凡结论是，$\alpha_2\left(\mu\right)$ 与 $\alpha_3\left(\mu\right)$ 的有效耦合常数确实在不违反质子寿命界的大统一能标处实现统一。

An account of two-loop corrections leads to the decrease of the M_{GUT} by factor 3. The parameter Δ in (102) is not small. Really, $\Delta/\left(\frac{1}{\alpha_2(M_{GUT})}\right)\approx 0.24$ and $\Lambda_{\Phi 1}\approx 2.3\cdot M_v$. It means that at the scale M_{GUT} , we must have some ultraviolet cutoff(regulator) to make sense to the nonrenormalizable interaction (101) at loop level. The promising way to deal with nonrenormalizable interactions is the use of nonlocal field theory. The simplest nonlocal generalization of the renormalizable Yang-Mills Lagrangian is given by the formula (59). Possible nonlocal generalization of nonrenormalizable interaction (101) is

考虑双圈修正后, M_{GUT} 缩小为原来的 1/3。(102) 中的参数 Δ 并不小, 事实上有 $\Delta/\left(\frac{1}{\alpha_2(M_{GUT})}\right)\approx 0.24$ 和 $\Lambda_{\Phi 1}\approx 2.3\cdot M_v$ 。这意味着在能标 M_{GUT} 处，我们必须引入紫外截断 (调节子)，才能让不可重整相互作用 (101) 在圈水平下具有意义。处理不可重整相互作用的可行方案是使用非局域场论。可重整杨-米尔斯拉格朗日量最简单的非局域推广由式 (59) 给出。不可重整相互作用 (101) 可能的非局域推广为

$$\Delta L_{F\Phi F\Phi,nl}=-\frac{1}{4\Lambda_{\Phi 1}^2}\left(\mathrm{Tr}\left(F_{\mu v}\Phi\right)V_{\Phi 1}\left(-\partial^\mu\partial_\mu\right)\mathrm{Tr}\left(F^{\mu v}\Phi\right)\right). \tag{104}$$

The use of nonlocal formfactors V and $V_{\Phi 1}$ cures bad ultraviot properties of nonrenormalizable interaction (104). For nonlocal Lagrangian (104), the parameter Δ depends on the scale μ :

使用非局域形状因子 V 和 $V_{\Phi 1}$ 可以解决不可重整相互作用 (104) 的不良紫外性质。对于非局域拉格朗日量 (104)，参数 Δ 依赖于能标 μ :

$$\Delta(\mu) = \frac{\pi\Phi_o^2}{\Lambda_{\Phi 1}^2} V_{\Phi 1}\left(-\mu^2\right) \tag{105}$$

We can use the normalization condition $V_{\Phi 1}\left(-M_{GUT}^2\right) = 1$. In this case formula (103) and numerical estimate for Δ are valid.

我们可以采用归一化条件 $V_{\Phi 1}\left(-M_{GUT}^2\right) = 1$ ，此时式 (103) 和对 Δ 的数值估计均成立。

Conclusions

结论

In this chapter we considered nonlocal quantum field theory including gauge fields and gravity. As the simplest example, we gave an overview of nonlocal scalar $d = 4\phi^4$ model. We have demonstrated that nonlocal ϕ^4 model is ultraviolet finite, unitary, and macrocausal. The main difference between renormalized ϕ^4 model and its nonlocal analog is that renormalized ϕ^4 model is local and microcausal while its nonlocal analog is only macrocausal. In other words the price of ultraviolet finiteness is the loss of locality and microcausality. Also we considered the model with an infinite number of local fields $\phi_n(x)$ and local interactions with higher-order derivatives for each local field. An account of infinite number of local fields leads to nonlocal and ultraviolet finite theory. So we can say that the presence of infinite number of local fields could be the origin of nonlocality. The nonlocal generalizations of abelian and nonabelian gauge theories are superrenormalizable field theories. From practical point of view, the main motivation that nonlocal field theory has to do with reality is the well-known fact that Einstein gravity is non-renormalizable theory at quantum level. Possible remedy to improve bad ultraviolet properties of Einstein gravity is the modification of gravity at Planck scale. In particular, nonlocal generalization of Einstein gravity leads to superrenormalizable theory. Also nonlocal renormalizable gravity model looks very promising. In this model ultraviolet behavior is determined by the Stelle model. The use of nonlocal formfactor allows to get rid of the problems with negative norm states for Stelle model. There is a very interesting question - what about the value of nonlocal scale? The most natural answer is that the nonlocal scale Λ coincides up to one or two orders with the quantum gravity scale $\varkappa^{-1} = 2.4 \cdot 10^{18}\mathrm{GeV}$. However we can’t exclude that $\Lambda \ll 2.4 \cdot 10^{18}\mathrm{GeV}$. As opposed to local renormalizable field theories, nonlocal field theories depend on unknown functions - formfactors. At present state of art, we can’t fix the form of nonlocal formfactor that strongly decreases the predictive power of nonlocal theory in comparison with renormalizable theory where the predictions depend on finite number of unknown parameters - masses and coupling constants. Also it is very interesting to mention that nonlocality can change drastically the classical solutions of Einstein gravity.

本章我们研究了包含规范场与引力的非定域量子场论。我们以最简单的例子概述了非定域标量 $d = 4\phi^4$ 模型，已经证明非定域 ϕ^4 模型是紫外有限、幺正且宏观因果的。重整化 ϕ^4 模型与其非定域类比模型的核心区别在于: 重整化 ϕ^4 模型是定域且微观因果的，而其非定域类比模型仅满足宏观因果性。换句话说，紫外有限性的代价是丧失定域性与微观因果性。我们还研究了包含无穷多个定域场 $\phi_n(x)$ ，且每个定域场都带有高阶导数定域相互作用的模型。引入无穷多定域场后可以得到非定域且紫外有限的理论，因此我们可以说，无穷多定域场的存在可能是非定域性的起源。阿贝尔与非阿贝尔规范理论的非定域推广都是超可重整化场论。从实用角度来看，非定域场论与现实相关的核心动机来自一个广为人知的事实: 爱因斯坦引力在量子层面是不可重整化的理论。改善爱因斯坦引力不良紫外性质的可行方案是在普朗克尺度对引力进行修改，具体而言，爱因斯坦引力的非定域推广可以得到超可重整化理论，而非定域可重整化引力模型也看起来非常有前景。该模型的紫外行为由施特勒模型确定，使用非定域形状因子可以解决施特勒模型的负规范态问题。有一个非常值得关注的问题: 非定域尺度的取值是多少？最自然的答案是，非定域尺度 Λ 与量子引力尺度 $\kappa^{-1} = 2.4 \cdot 10^{18}\mathrm{GeV}$ 相差仅一到两个数量级。不过我们也不能排除 $\Lambda \ll 2.4 \cdot 10^{18}\mathrm{GeV}$ 的可能性。与定域可重整化场论不同，非定域场论依赖于未知函数——形状因子。在当前的研究水平下，我们无法确定非定域形状因子的形式，这导致非定域理论的预言能力远低于可重整化理论——后者的预言仅依赖有限个未知参数: 质量与耦合常数。另外非常值得一提的是，非定域性可以彻底改变爱因斯坦引力的经典解。

There are several interesting applications of nonlocal field theory. In particular, in this chapter we considered the models with γ_5 -anomalies. As is well known the models with γ_5 -anomalies spoil gauge invariance at quantum level. As a consequence models with γ_5 -anomalies are nonrenormalizable. On the example of axial QED, we have demonstrated that the introduction of nonlocality allows to make axial QED superrenormalizable. Nonlocal axial QED describes the interaction of massive vector photon with massless fermions. Also we considered nonlocal generalization of Georgi-Glashow $SU(5)$ GUT. An account of nonlocality can cure Georgi-Glashow $SU(5)$ GUT with its wrong phenomenological predictions for proton decay and the Weinberg angle θ_W . In the simplest nonlocal extension of the Georgi-Glashow $SU(5)$ GUT, the value of GUT scale is $M_{GUT} \approx 3 \cdot 10^{16}\mathrm{GeV}$. The nonlocal scale Λ in this model coincides by the order of magnitude with the GUT scale M_{GUT} , and it is by two orders of magnitude smaller the quantum gravity scale $\kappa^{-1} = 2.4 \cdot 10^{18}\mathrm{GeV}$.

非定域场论已有若干有趣的应用。具体而言，本章我们研究了带有 γ_5 反常的模型。众所周知，带有 γ_5 反常的模型会在量子层面破坏规范不变性，因此带 γ_5 反常的模型都是不可重整化的。我们以轴矢量量子电动力学为例证明了，引入非定域性可以使轴矢量量子电动力学变为超可重整化理论。非定域轴矢量量子电动力学描述了有质量矢量光子与无质量费米子的相互作用。我们还研究了乔治-格拉肖 $SU(5)$ 大统一理论的非定域推广。引入非定域性可以解决乔治-格拉肖 $SU(5)$ 大统一理论在质子衰变和温伯格角 θ_W 上错误的唯象预言问题。在乔治-格拉肖 $SU(5)$ 大统一理论最简单的非定域扩展中，大统一统一尺度的值为 $M_{GUT} \approx 3 \cdot 10^{16}\mathrm{GeV}$ 。该模型中的非定域尺度 Λ 与大统一尺度 M_{GUT} 量级一致，且比量子引力尺度 $\kappa^{-1} = 2.4 \cdot 10^{18}\mathrm{GeV}$ 小两个数量级。

I am indebted to the collaborators of the INR theoretical department for discussions and critical comments.

感谢 INR 理论部的合作者参与讨论并提出建设性批评意见。

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